Algebra 2 Second Semester Review

Multiple Choice
*Identify the choice that best completes the statement or answers the question.*

____ 1. A restaurant had a lot of customers for breakfast and dinner, but not many customers for lunch. Which graph best represents the number of customers in the restaurant that day?

a. ![Graph A]
   
   b. ![Graph B]
   
   c. ![Graph C]
   
   d. ![Graph D]

Short Answer

1. The table shows the probability distribution for the number of people who contract a disease in a scientific study. Find the expected number of people who contract the disease. Round your answer to the nearest tenth.

<table>
<thead>
<tr>
<th>Number of People</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.20</td>
<td>0.32</td>
<td>0.288</td>
<td>0.153</td>
<td>0.0384</td>
</tr>
</tbody>
</table>
2. The number of calls received by a technical support center during 18 randomly selected days is listed. Identify the outlier, and describe how it affects the mean and the standard deviation.

<table>
<thead>
<tr>
<th>50</th>
<th>57</th>
<th>77</th>
<th>66</th>
<th>53</th>
<th>72</th>
</tr>
</thead>
<tbody>
<tr>
<td>51</td>
<td>88</td>
<td>82</td>
<td>70</td>
<td>62</td>
<td>64</td>
</tr>
<tr>
<td>69</td>
<td>88</td>
<td>98</td>
<td>65</td>
<td>14</td>
<td>68</td>
</tr>
</tbody>
</table>

3. An oil company plans to add a chemical to its gasoline to make it burn more cleanly. The company conducts an experiment to see whether adding the chemical affects the gasoline mileage of cars using their gasoline. State the null hypothesis for the experiment.

4. A speed reading course claims that it can boost reading speeds to 1050 words per minute. In a random sample of 49 people who took the course, the average was 1020 words per minute, with a standard deviation of 90 words per minute. What is the z-value rounded to the nearest hundredth? Is there enough evidence to reject the claim?

5. Voters in Jackson County are going to vote on a half-percent sales tax increase to support music in local schools. According to a random survey, 40% plan to vote for the tax and 60% plan to vote against it. The survey’s margin of error is ±6%. Determine whether the survey clearly projects whether the sales tax will pass. Explain your response.

6. Use the Binomial Theorem to expand the binomial \((2x - 4y)^4\).

7. Students randomly receive 1 of 4 versions (A, B, C, D) of a math test. What is the probability that at least 3 of the 5 students tested will get version A of the test? Express your answer as a percent, and round to the nearest tenth.

8. Suppose \(x\) is a normally-distributed random variable with mean \(\mu = 25\) and standard deviation \(\sigma = 2\). Use the table to find the probability that \(x > 22\). Express your answer as a decimal.

<table>
<thead>
<tr>
<th>(z)</th>
<th>-2.5</th>
<th>-2</th>
<th>-1.5</th>
<th>-1</th>
<th>-0.5</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area</td>
<td>0.01</td>
<td>0.02</td>
<td>0.07</td>
<td>0.16</td>
<td>0.31</td>
<td>0.5</td>
<td>0.69</td>
<td>0.84</td>
<td>0.93</td>
<td>0.98</td>
<td>0.99</td>
</tr>
</tbody>
</table>

9. At a school carnival, you can win tickets to trade for prizes. A particular game has 5 possible outcomes. What is the expected number of tickets won?

<table>
<thead>
<tr>
<th>Tickets won</th>
<th>18</th>
<th>38</th>
<th>45</th>
<th>70</th>
<th>91</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.32</td>
<td>0.26</td>
<td>0.19</td>
<td>0.14</td>
<td>0.09</td>
</tr>
</tbody>
</table>
10. Joyce won $400 in an essay contest. She invests the money in an interest-earning account. The table shows how much money she has in the account. Find an appropriate model for the amount that Joyce will have in the account after $t$ years. Then, use the model to predict approximately when Joyce will have $800 in the account.

<table>
<thead>
<tr>
<th>Year</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$424.00</td>
</tr>
<tr>
<td>2</td>
<td>$449.44</td>
</tr>
<tr>
<td>3</td>
<td>$476.41</td>
</tr>
<tr>
<td>4</td>
<td>$504.99</td>
</tr>
<tr>
<td>5</td>
<td>$535.29</td>
</tr>
</tbody>
</table>

11. Find the end behavior of the function $P(x) = -7x^4$.

12. Find the end behavior of the function $P(x) = 18x^6$.

13. Find the end behavior of the function $P(x) = -3x^5$.

14. Find the end behavior of the function $P(x) = \frac{1}{3}x^2$.

15. Write an equation that represents the graph below.
16. Evaluate the piecewise function \( f(x) = \begin{cases} 11 & \text{if } x \leq 5 \\ -14 & \text{if } 5 < x \leq 6 \\ 1 & \text{if } 6 < x \end{cases} \) for \( x = -1 \) and \( x = 9 \).

\[
\begin{align*}
5x^3 - 2 & \text{ if } x < -6 \\
7 + 2x & \text{ if } x \geq 7 \\
\end{align*}
\]

17. Evaluate \( f(-3) \) if \( f(x) = \begin{cases} 3x^2 - 2 & \text{if } -6 \leq x < 7 \\ 7 + 2x & \text{if } x \geq 7 \end{cases} \).

\[
\begin{align*}
6x^3 - 2 & \text{ if } x < -9 \\
2x^2 - 10 & \text{ if } -9 \leq x < 7 \\
4 + 2x & \text{ if } x \geq 7 \\
\end{align*}
\]

18. Evaluate \( f(6) \) if \( f(x) = \begin{cases} 3x^2 - 2 & \text{if } -6 \leq x < 7 \\ 7 + 2x & \text{if } x \geq 7 \end{cases} \).

19. Graph the piecewise function \( h(x) = \begin{cases} -7 & \text{if } x < 4 \\ 7 & \text{if } x \geq 4 \end{cases} \).

20. Given \( f(x) = \begin{cases} 2x^2 + 1 & \text{if } x > 0 \\ -x + 1 & \text{if } x \leq 0 \end{cases} \), write the rule for \( g(x) \), a horizontal translation of \( f(x) \) 4 units to the left.

21. The price of dance lessons depends upon the number of lessons that you select. If \( x \) is the number of lessons then the fee for the lessons (in dollars) can be found using the piecewise function

\[
\begin{align*}
40x & \text{ if } 0 < x \leq 4 \\
30x & \text{ if } 4 < x \leq 8 \\
25x & \text{ if } x > 8 \\
\end{align*}
\]

The lessons are increasing by 10% per lesson with a $5 processing fee for each student. What is the new function for the cost of lessons?

22. Given \( f(x) = 2x^2 + 8x - 4 \) and \( g(x) = -5x + 6 \), find \( (f - g)(x) \).

23. Given \( f(x) = 4x^2 + 3x - 5 \) and \( g(x) = -2x + 12 \), find \( (fg)(x) \).

24. Given \( f(x) = x^3 \) and \( g(x) = 4x + 3 \), find \( g(f(3)) \).

25. Find the inverse of \( f(x) = (3x - 24)^4 \). Determine whether it is a function, and state its domain and range.
26. Determine by composition whether \( f(x) = \frac{1}{5}x + 4 \) and \( g(x) = 5x - 20 \) are inverses.

27. Louise wears an outfit everyday that consists of one top (shirt, T-shirt, or blouse), one bottom (pants or skirt) and one scarf. Her wardrobe consists of a tan skirt, a pair of black pants, 2 T-shirts, one silk blouse, 1 button-down shirt, and a set of 3 scarves. How many different outfits can Louise put together?

28. There are 7 singers competing at a talent show. In how many different ways can the singers appear?

29. Joel owns 12 shirts and is selecting the ones he will wear to school next week. How many different ways can Joel choose a group of 5 shirts? (Note that he will not wear the same shirt more than once during the week.)

30. An experiment consists of rolling a number cube. What is the probability of rolling a number greater than 4? Express your answer as a fraction in simplest form.

31. A person is selected at random. What is the probability that the person was not born on a Monday? Express your answer as a percent. If necessary, round your answer to the nearest tenth of a percent.

32. In a recent survey of 25 voters, 17 favor a new city regulation and 8 oppose it. What is the probability that in a random sample of 6 respondents from this survey, exactly 2 favor the proposed regulation and 4 oppose it? Express your answer as a decimal.

33. A circle is inscribed in a square with a side length of 4. If a point in the square is chosen at random, what is the probability that the point is in the square but not in the circle? Express your answer as a percent, and round to the nearest tenth.
34. An experiment consists of spinning a spinner. The table shows the results. Find the experimental probability that the spinner does not land on red. Express your answer as a fraction in simplest form.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>red</td>
<td>10</td>
</tr>
<tr>
<td>purple</td>
<td>11</td>
</tr>
<tr>
<td>yellow</td>
<td>13</td>
</tr>
</tbody>
</table>

35. A bag contains hair ribbons for a spirit rally. The bag contains 3 black ribbons and 17 green ribbons. Lila and Jessica are drawing from the bag at random. Find the indicated probability.

Lila selects a black ribbon and then Jessica selects a green ribbon.

36. The table shows the distribution of the labor force in the United States in the year 2000. Suppose that a worker is selected at random. Find the probability that a female works in the Industry field. Express your answer as a decimal, and round to the nearest thousandth.

<table>
<thead>
<tr>
<th></th>
<th>Agriculture</th>
<th>Industry</th>
<th>Services</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>3,132,000</td>
<td>25,056,000</td>
<td>50,112,000</td>
</tr>
<tr>
<td>Female</td>
<td>667,000</td>
<td>8,004,000</td>
<td>57,362,000</td>
</tr>
</tbody>
</table>

37. A grab bag contains 8 football cards and 2 basketball cards. An experiment consists of taking one card out of the bag, replacing it, and then selecting another card. Determine whether the events are independent or dependent. What is the probability of selecting a football card and then a basketball card? Express your answer as a decimal.

38. A poll of 100 senior citizens in a retirement community asked about the types of electronic communication they used. The table shows the joint and marginal frequencies from the poll results. If you are given that one of the people polled uses text messaging, what is the probability that the person is also using e-mail? Express your answer as a decimal. If necessary, round your answer to the nearest hundredth.

<table>
<thead>
<tr>
<th>Uses text messaging</th>
<th>Yes</th>
<th>No</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uses e-mail</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yes</td>
<td>0.17</td>
<td>0.66</td>
<td>0.83</td>
</tr>
<tr>
<td>No</td>
<td>0.11</td>
<td>0.06</td>
<td>0.17</td>
</tr>
<tr>
<td>Total</td>
<td>0.28</td>
<td>0.72</td>
<td>1</td>
</tr>
</tbody>
</table>
39. At a small high school, there are 80 girls in the senior class. Some of them play basketball, some play soccer, some play both, and some play neither. The table shows the joint and marginal frequencies for the senior girls.
If you know that a girl plays soccer, what is the probability that she also plays basketball? Express your answer as a decimal. If necessary, round your answer to the nearest hundredth.

<table>
<thead>
<tr>
<th>Plays basketball</th>
<th>Yes</th>
<th>No</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plays soccer</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yes</td>
<td>0.075</td>
<td>0.100</td>
<td>0.175</td>
</tr>
<tr>
<td>No</td>
<td>0.250</td>
<td>0.575</td>
<td>0.825</td>
</tr>
<tr>
<td>Total</td>
<td>0.325</td>
<td>0.675</td>
<td>1</td>
</tr>
</tbody>
</table>

40. Find the probability of rolling a 5 or an odd number on a number cube. Express your answer as a fraction in simplest form.

41. Find the first 5 terms of the sequence with \( a_1 = 6 \) and \( a_n = 2a_{n-1} - 1 \) for \( n \geq 2 \).

42. Find the first 5 terms of the sequence \( a_n = 2^n - 5 \).

43. Write a possible explicit rule for \( n \)th term of the sequence 23.1, 20.2, 17.3, 14.4, 11.5, 8.6, ...

44. A small island in the middle of a river is eroding away. Each year, the island has 85% of the area from the previous year. After one year the island has an area of 10.2 thousand square yards. Graph the sequence and describe the pattern. How much of the island is left after 6 years?

45. There are 3 squares of increasing size. The ratio of a side of one square to a side of the next larger square is \( \frac{1}{\sqrt{2}} \). The first square has an area of 9 square units. Find the area of the third square.

46. Write the series \(-\frac{1}{2} + \frac{1}{4} - \frac{1}{6} + \frac{1}{8} - \frac{1}{10} + \frac{1}{12}\) in summation notation.

47. Expand the series \( \sum_{k=2}^{6} (-1)^k (7-k)k \) and evaluate.

48. Evaluate the series \( \sum_{k=1}^{22} k \).

49. Determine whether the sequence \(-1, 7, 15, 23, 31, \ldots\) could be an arithmetic sequence. If so, find the common difference and the next three terms in the sequence.
50. Find the 22nd term in the arithmetic sequence \(-5, -9, -13, -17, -21, \ldots\)

51. Find the 5th term of the arithmetic sequence with \(a_7 = 25\) and \(a_{13} = 55\).

52. Find the sum for the arithmetic series \(\sum_{k=1}^{13} 15k - 4\).

53. Write the arithmetic series \(5 + 1 - 3 - 7 - 11 - 15 - 19\) in summation notation.

54. Find the 7th term of the geometric sequence \(-4, 12, -36, 108, -324, \ldots\)

55. Find the 7th term of the geometric sequence with \(a_3 = 16\) and \(a_5 = 64\).

56. Find the sum \(S_8\) for the geometric series \(6 + 0.6 + 0.06 + 0.006 + \ldots\)

57. Find the first 3 terms of the geometric sequence with \(a_6 = -128\) and \(a_{11} = 4,096\).

58. Find the value of the sine, cosine, and tangent functions for \(\theta\) where \(A = 96\), \(B = 28\), and \(C = 100\).

59. After takeoff from an airport, an airplane’s angle of ascent is \(10^\circ\). The airplane climbs to an altitude of 10,000 feet. At that point, what is the land distance between the airplane and the airport? Round your answer to the nearest foot.

60. A surveyor whose eye level is 5 feet above the ground determines the angle of elevation to the top of an office building to be \(41.7^\circ\). If the surveyor is standing 40 feet from the base of the building, what is the height of the building to the nearest foot?
61. Find the values of the six trigonometric functions for $\theta$.

![Diagram showing a right triangle with sides 14, 48, and $c$]

62. Convert $\frac{17\pi}{10}$ from radians to degrees.

63. Convert $-35^\circ$ to radians.

64. Use the unit circle to find the exact value of the trigonometric function $\cos 30^\circ$.

65. A bicycle tire with a diameter of 26 inches makes 3 revolutions per second. To the nearest mile, how far does a point on the edge of the tire travel in one hour? (Hint: Find how far the bicycle travels in one hour.)

66. A 35-foot telephone pole casts a 46-foot shadow on the ground while the sun is shining. To the nearest degree, what is the angle of elevation of the sun from the end of the shadow?

![Diagram showing the shadow of a telephone pole]

67. You can use trigonometry to measure the height of a pyramid in Egypt.

[1.] An archaeologist positions himself 260 ft from the base of a pyramid so that his eye level is 5 ft above the ground. If the pyramid is 500 feet in height, what would be the angle of elevation from the archaeologist to the top of the pyramid?

[2.] The angle of elevation from the eye level of an archaeologist to the top of a pyramid whose base is 400 feet away is $50^\circ$. To the nearest foot, what is the height of the pyramid?
68. Solve the triangle. \( m \angle N = 118^\circ \), \( m \angle P = 33^\circ \), and \( m = 15 \). Round to the nearest tenth.

\[
\begin{array}{c}
N \\
p \\
m \\
M \\
n \\
P
\end{array}
\]

69. Adolfo surveyed a triangular region of land and sent the measurements: \( a = 130 \) meters, \( b = 150 \) meters, \( m \angle A = 49^\circ \) to the engineer at the office. The engineer called back asking for more information. Determine how many, if any, triangles can be formed using Adolfo’s measurements. If possible, find the unknown measurements.

70. To measure the width of a river, a ranger sights a tree across the river about 60° north of east. On the map, her position is marked \( A \). Then, she walks along the river to position \( B \) and sights the same tree about 40° north of west. If each unit on the map represents 1 mile, about how many miles from her current position is the tree?

\[
\begin{array}{c}
\text{\( x \)} \\
\text{\( y \)} \\
-2 & -1 & 1 & 2 & 3 & 4 & 5 & 6 \\
-1 & -2 & 1 & 2 & 3 & 4 & 5 & 6 \\
A & B
\end{array}
\]

71. Identify the conic section \( 16x^2 + 4y^2 = 64 \), then describe its properties (center, vertices, co-vertices, foci, and/or directrix)

72. Identify the conic section \( y^2 - x^2 = 81 \), then describe its properties (center, vertices, co-vertices, foci, and/or directrix).

73. Find the center and radius of a circle that has a diameter with endpoints \((-9, -6)\) and \((-1, 0)\).

74. Write the equation of a circle with center \((8, 7)\) and radius \( r = 6 \).
75. Write an equation in standard form for the ellipse shown with center \((0, 0)\).

![Ellipse Diagram]

76. Graph the ellipse \(\frac{(x-6)^2}{100} + \frac{(y+5)^2}{64} = 1\).

77. The path that a satellite travels around Earth is an ellipse with Earth at one focus. The length of the major axis is about 16,000 km, and the length of the minor axis is about 12,000 km. Write an equation for the satellite’s orbit.

78. Write an equation in standard form for the hyperbola with center \((0, 0)\), vertex \((0, 6)\), and focus \((0, 8)\).

79. Find the vertices, co-vertices, and asymptotes of the hyperbola \(\frac{(y-1)^2}{25} - \frac{(x+2)^2}{9} = 1\), and then graph.

80. Write the equation in standard form for the parabola with vertex \((0,0)\) and directrix \(y = -14\).

81. Identify the conic section that the equation \(\frac{(x-2)^2}{3^2} + \frac{(y-4)^2}{7^2} = 1\) represents.

82. Identify the conic section that the equation \((x+9)^2 + (y-6)^2 = 48\) represents.

83. Identify the conic section that the equation \(y^2 - 6y + 9 - x = 5\) represents.

84. Identify the conic section that the equation \(7x^2 - 8xy - 4y^2 - 7x - 6y + 10 = 0\) represents.
85. Solve \[
\begin{align*}
x^2 + y^2 &= 25 \\
y + 5 &= \frac{1}{2}x^2
\end{align*}
\]

86. Solve \[
\begin{align*}
4x^2 + 9y^2 &= 36 \\
4x^2 - 25y^2 &= 100
\end{align*}
\]

87. Two balls are rolling on a table top. The path of one ball can be modeled by the equation \(x^2 + 4y^2 = 7\). The path of the other ball can be modeled by the equation \(2y^2 - x = 4\). Find the coordinates of the points where the paths intersect.

88. A scientist studies a herd of mule deer to learn about their dietary habits. Identify the population and sample.

89. A social networking website surveys its members to find out their opinion on its privacy safeguards. They send an e-mail to all members, hoping that at least 10% of them respond. Classify the sample.

90. In the cafeteria, sometimes salads are served and sometimes fruit is served. Linda notes that out of 15 days, 12 days salad is served and 3 days fruit is served. Predict how many days fruit is served in a 180-day school year. If necessary, round your answer to the nearest whole number.
Algebra 2 Second Semester Review
Answer Section

MULTIPLE CHOICE

1. ANS: A
   The graph shows more customers in the restaurant in the morning and evening, but fewer customers in the middle of the day.

<table>
<thead>
<tr>
<th>Feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Correct!</td>
</tr>
<tr>
<td>B This graph has more customers at lunch than at breakfast or dinner.</td>
</tr>
<tr>
<td>C This graph has more customers at lunch than at breakfast or dinner.</td>
</tr>
<tr>
<td>D This graph shows more customers at lunch than at breakfast.</td>
</tr>
</tbody>
</table>

   PTS: 1 DIF: Basic REF: 17081b46-4683-11df-9c7d-001185f0d2ea
   OBJ: 6-1.1 Application NAT: NT.CCSS.MTH.10.9-12.F.IF.4
   LOC: MTH.C.10.07.01.01.003 TOP: 6-1 Multiple Representations of Functions
   MSC: DOK 2

SHORT ANSWER

1. ANS:
   3.5
   The expected value is the weighted average of all the outcomes of the study.

   Expected value = 2(.20) + 3(.32) + 4(.288) + 5(.1536) + 6(.0384) = 3.5104 ≈ 3.5

   PTS: 1 DIF: Average REF: 17aa3bb6-4683-11df-9c7d-001185f0d2ea
   OBJ: 8-1.2 Finding Expected Value NAT: NT.CCSS.MTH.10.9-12.S.MD.5 | NT.CCSS.MTH.10.9-12.S.MD.4
   LOC: MTH.C.13.05.05.004 TOP: 8-1 Measures of Central Tendency and Variation
   MSC: DOK 3

2. ANS:
   The outlier is 14. The outlier in the data set causes the mean to decrease from about 69.4 to about 66.3 and the standard deviation to increase from about 13.7 to about 18.6.

   The outlier is 14. The outlier in the data set causes the mean to decrease from about 69.4 to about 66.3 and the standard deviation to increase from about 13.7 to about 18.6.

   PTS: 1 DIF: Average REF: 17af006e-4683-11df-9c7d-001185f0d2ea
   OBJ: 8-1.5 Examining Outliers NAT: NT.CCSS.MTH.10.9-12.S.ID.3
   LOC: MTH.C.13.04.02.01.009 | MTH.C.13.04.02.013 | MTH.C.13.04.02.02.014
   TOP: 8-1 Measures of Central Tendency and Variation MSC: DOK 3
3. **ANS:**
Adding the chemical does not affect gasoline mileage.

**PTS:** 1  
**DIF:** Average  
**REF:** 908efba2-6ab2-11e0-9e90-001185f0d2ea

**OBJ:** 8-4.1 Analyzing a Controlled Experiment  
**NAT:** NT.CCSS.MTH.10.9-12.S.IC.5

**TOP:** 8-4 Significance of Experimental Results  
**KEY:** significance | experiment

**MSC:** DOK 3

4. **ANS:**
The z-value is $-2.33$.
There is enough evidence to reject the claim.

**PTS:** 1  
**DIF:** Average  
**REF:** 9091850d-6ab2-11e0-9c90-001185f0d2ea

**OBJ:** 8-4.2 Using a z-Test  
**NAT:** NT.CCSS.MTH.10.9-12.S.IC.5

**TOP:** 8-4 Significance of Experimental Results  
**KEY:** significance | experiment | z-test

**MSC:** DOK 3

5. **ANS:**
The survey clearly projects that the sales tax will not pass; $40\% \pm 6\% = 34\%$ to $46\%$ plan to vote for the tax and $60\% \pm 6\% = 54\%$ to $66\%$ plan to vote against the tax. The intervals do not overlap, so the survey clearly projects the outcome.

**PTS:** 1  
**DIF:** Average  
**REF:** 909d49c4-6ab2-11e0-9c90-001185f0d2ea

**OBJ:** 8-5.3 Interpreting a Margin of Error  
**NAT:** NT.CCSS.MTH.10.9-12.S.IC.4 | NT.CCSS.MTH.10.9-12.S.IC.6

**TOP:** 8-5 Sampling Distributions  
**KEY:** survey | margin of error

**MSC:** DOK 3

6. **ANS:**

$16x^4 - 128x^3y + 384x^2y^2 - 512xy^3 + 256y^4$

Use Pascal’s Triangle (or the combinations used to derive the triangle) to help determine the coefficients for each term in the expansion:

$\binom{4}{0}(2x)^4(-4y)^0 + \binom{4}{1}(2x)^3(-4y)^1 + \binom{4}{2}(2x)^2(-4y)^2 + \binom{4}{3}(2x)^1(-4y)^3 + \binom{4}{4}(2x)^0(-4y)^4$

Calculate the combinations:

$1 \times 16x^4 \times 1 + 4 \times 8x^3 \times (-4)y + 6 \times 4x^2 \times 16y^2 + 4 \times 2x \times (-64)y^3 + 1 \times 1 \times 256y^4$

Simplify:

$16x^4 - 128x^3y + 384x^2y^2 - 512xy^3 + 256y^4$

**PTS:** 1  
**DIF:** Average  
**REF:** 17b39e16-4683-11df-9c7d-001185f0d2ea

**OBJ:** 8-6.1 Expanding Binomials  
**NAT:** NT.CCSS.MTH.10.9-12.A.APR.5

**STA:** CA.CACS.MTH.97.ALI2.AII.20.0  
**LOC:** MTH.C.10.05.08.03.01.01.001

**TOP:** 8-6 Binomial Distributions  
**MSC:** DOK 3
7. ANS:

10.4%
The probability that a student will receive version A of the test is \( \frac{1}{4} \), or 0.25.

\[ P(r) = \binom{n}{r} p^r q^{n-r} \]

At least 3 students is the same as exactly 3, 4, or 5 students receiving version A of the test.

\[ P(\geq 3 \text{ students receiving test version A}) = P(3) + P(4) + P(5) \]

\[ = \binom{5}{3} (0.25)^3 (0.75)^{5-3} + \binom{5}{4} (0.25)^4 (0.75)^{5-4} + \binom{5}{5} (0.25)^5 (0.75)^{5-5} \]

\[ = 10(0.015625)(0.5625) + 5(0.00390625)(0.75) + 1(0.0009765625)(1) \]

\[ = 0.087890625 + 0.0146484375 + 0.0009765625 \]

\[ \approx 0.1035 \]

The probability that at least 3 students will get version A of the test is 0.1035, or about 10.4%.

PTS: 1    DIF: Average    REF: 17b3c526-4683-11df-9c7d-001185f0d2ea
OBJ: 8-6.2 Finding Binomial Probabilities    LOC: MTH.C.13.05.04.005
TOP: 8-6 Binomial Distributions    MSC: DOK 3

8. ANS:

0.93

PTS: 1    DIF: Average    REF: 90a6d330-6ab2-11e0-9c90-001185f0d2ea
NAT: NT.CCSS.MTH.10.9-12.S.ID.4    TOP: 8-7 Fitting to a Normal Distribution
KEY: normal distribution | z-value    MSC: DOK 3

9. ANS:

42.18

PTS: 1    DIF: Average    REF: 90ab97e6-6ab2-11e0-9c90-001185f0d2ea
OBJ: 8-8.2 Using Expected Value in Real-World Situations    TOP: 8-8 Analyzing Decisions
KEY: analyzing decisions | expected value    MSC: DOK 3
10. **ANS:**

\[ V(t) = 400(1.06)^t; \text{ about 12 years} \]

**Step 1**

The investment grows at an exponential rate.

First, divide the amount in one year by the amount in the previous year.

\[
\text{Value in year 3} = \frac{476.41}{449.44} \approx 1.06
\]

Then, multiply the result by the initial amount, $400.

So \( V(t) = 400(1.06)^t \).

**Step 2**

Evaluate the function for different values of \( t \) to find when Joyce will have $800 in the account.

\[ V(11) = 400(1.06)^{11} \approx 759.32 \]

\[ V(12) = 400(1.06)^{12} \approx 804.88 \]

So Joyce will have $800 in approximately 12 years.

PTS: 1  DIF: Advanced  REF: 170cddf1-4683-11df-9e7d-001185f0d2ea
NAT: NT.CCSS.MTH.10.9-12.A.CED.2 | NT.CCSS.MTH.10.9-12.F.LE.2
LOC: MTH.C.13.04.04.002 | MTH.C.13.04.04.010
TOP: 6-1 Multiple Representations of Functions  MSC: DOK 3

11. **ANS:**

As \( x \to -\infty \), \( P(x) \to \infty \) and as \( x \to +\infty \), \( P(x) \to \infty \)

PTS: 1  DIF: Basic  REF: 9068d5f2-6ab2-11e0-9e70-001185f0d2ea
OBJ: 6-2.3 Comparing Exponential and Polynomial Functions
NAT: NT.CCSS.MTH.10.9-12.F.IF.7.c  TOP: 6-2 Comparing Functions
KEY: end behavior | polynomial functions  MSC: DOK 2

12. **ANS:**

As \( x \to -\infty \), \( P(x) \to +\infty \) and as \( x \to +\infty \), \( P(x) \to +\infty \)

PTS: 1  DIF: Basic  REF: 9068d5f2-6ab2-11e0-9e70-001185f0d2ea
OBJ: 6-2.3 Comparing Exponential and Polynomial Functions
NAT: NT.CCSS.MTH.10.9-12.F.IF.7.c  TOP: 6-2 Comparing Functions
KEY: end behavior | polynomial functions  MSC: DOK 2

13. **ANS:**

As \( x \to -\infty \), \( P(x) \to \infty \) and as \( x \to +\infty \), \( P(x) \to \infty \)

PTS: 1  DIF: Basic  REF: 9068d5f2-6ab2-11e0-9e70-001185f0d2ea
OBJ: 6-2.3 Comparing Exponential and Polynomial Functions
NAT: NT.CCSS.MTH.10.9-12.F.IF.7.c  TOP: 6-2 Comparing Functions
KEY: end behavior | polynomial functions  MSC: DOK 2
14. ANS:
As\( x \to -\infty \), \( P(x) \to -\infty \) and as \( x \to +\infty \), \( P(x) \to -\infty \)

PTS: 1  DIF: Basic  REF: 9068d5f2-6ab2-11e0-9c90-001185f0d2ea  
OBJ: 6-2.3 Comparing Exponential and Polynomial Functions  
NAT: NT.CCSS.MTH.10.9-12.F.IF.7.c TOP: 6-2 Comparing Functions  
KEY: end behavior | polynomial functions  
MSC: DOK 2

15. ANS:

<table>
<thead>
<tr>
<th>Weight (lbs)</th>
<th>Shipping Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 1.0</td>
<td>3.85</td>
</tr>
<tr>
<td>1.1 – 2.0</td>
<td>4.55</td>
</tr>
<tr>
<td>2.1 – 3.0</td>
<td>6.05</td>
</tr>
<tr>
<td>3.1 – 4.0</td>
<td>7.05</td>
</tr>
<tr>
<td>4.1 – 5.0</td>
<td>8.00</td>
</tr>
</tbody>
</table>

The cost to ship a package Priority Mail is $3.85 for packages not over 1.0 pound, $4.55 for packages weighing 1.1 to 2.0 pounds, $6.05 for packages weighing 2.1 to 3.0 pounds, $7.05 for packages weighing 3.1 to 4.0 pounds, and $8.00 for packages weighing 4.1 to 5.0 pounds. The domain of the function is divided into five intervals:

<table>
<thead>
<tr>
<th>Interval</th>
<th>Verbal Description of Interval</th>
<th>Shipping Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0, 1]</td>
<td>not over 1.0 pound</td>
<td>3.85</td>
</tr>
<tr>
<td>(1, 2]</td>
<td>1.1 to 2.0 pounds</td>
<td>4.55</td>
</tr>
<tr>
<td>(2, 3]</td>
<td>2.1 to 3.0 pounds</td>
<td>6.05</td>
</tr>
<tr>
<td>(3, 4]</td>
<td>3.1 to 4.0 pounds</td>
<td>7.05</td>
</tr>
<tr>
<td>(4, 5]</td>
<td>4.1 to 5.0 pounds</td>
<td>8.00</td>
</tr>
</tbody>
</table>

PTS: 1  DIF: Average  REF: 170f1b4a-4683-11df-9c7d-001185f0d2ea  
OBJ: 6-3.1 Application  NAT: NT.CCSS.MTH.10.9-12.F.IF.4  
TOP: 6-3 Piecewise Functions  MSC: DOK 3

16. ANS:
\( f(-1) = 11; f(9) = 1 \)

\( f(-1) = 11 \) Because \(-1 \leq x \leq 5\), use the rule for \( x \leq 5 \).

\( f(9) = 1 \) Because \( 6 \leq 9 \), use the rule for \( 6 < x \).

PTS: 1  DIF: Basic  REF: 170f425a-4683-11df-9c7d-001185f0d2ea  
OBJ: 6-3.2 Evaluating a Piecewise Function  NAT: NT.CCSS.MTH.10.9-12.F.IF.2  
LOC: MTH.C.10.07.04.01.005  TOP: 6-3 Piecewise Functions  
MSC: DOK 1
17. ANS:
106

PTS: 1  DIF: Average  REF: 1730a362-4683-11df-9c7d-001185f0d2ea
NAT: NT.CCSS.MTH.10.9-12.F.IF.2  LOC: MTH.C.10.07.04.01.005
TOP: 6-3 Piecewise Functions  MSC: DOK 1

18. ANS:
152

PTS: 1  DIF: Average  REF: 1730a362-4683-11df-9c7d-001185f0d2ea
NAT: NT.CCSS.MTH.10.9-12.F.IF.2  LOC: MTH.C.10.07.04.01.005
TOP: 6-3 Piecewise Functions  MSC: DOK 1

19. ANS:

The function has two constant pieces that will be represented by horizontal rays.
Because the domain is divided at $x = 4$, evaluate both branches of the function at $x = 4$.
The function is $-7$ when $x < 4$, so plot the point $(4, -7)$ with an open circle and draw a horizontal ray to the left.
The function is $7$ when $x \geq 4$, so plot the point $(4, 7)$ with a solid dot and draw a horizontal ray to the right.

PTS: 1  DIF: Basic  REF: 17117da6-4683-11df-9c7d-001185f0d2ea
OBJ: 6-3.3 Graphing Piecewise Functions  NAT: NT.CCSS.MTH.10.9-12.F.IF.7.b
LOC: MTH.C.10.07.04.01.006  TOP: 6-3 Piecewise Functions
MSC: DOK 2
20. ANS:

\[ f(x) = \begin{cases} 
2x^2 + 16x + 33 & \text{if } x > -4 \\
-x - 3 & \text{if } x \leq -4
\end{cases} \]

Each piece of \( f(x) \) must be shifted 4 units to the left. Replace every \( x \) with \((x + 4)\), and simplify.

\[ g(x) = f(x + 4) = \begin{cases} 
2(x + 4)^2 + 1 & \text{if } (x + 4) > 0 \\
-(x + 4) + 1 & \text{if } (x + 4) \leq 0
\end{cases} \]

\[ g(x) = f(x + 4) = \begin{cases} 
2(x^2 + 8x + 16) + 1 & \text{if } x > -4 \\
-x - 4 + 1 & \text{if } x \leq -4
\end{cases} \]

\[ g(x) = f(x + 4) = \begin{cases} 
2x^2 + 16x + 33 & \text{if } x > -4 \\
-x - 3 & \text{if } x \leq -4
\end{cases} \]

PTS: 1 DIF: Average REF: 17140712-4683-11df-9c7d-001185f0d2ea
OBJ: 6-4.1 Transforming Piecewise Functions NAT: NT.CCSS.MTH.10.9-12.F.BF.3
LOC: MTH.C.10.07.16.02.003 TOP: 6-4 Transforming Functions
MSC: DOK 3

21. ANS:

\[ f(x) = \begin{cases} 
44x + 5 & \text{if } 0 < x \leq 4 \\
33x + 5 & \text{if } 4 < x \leq 8 \\
27.5x + 5 & \text{if } x > 8
\end{cases} \]

The fee per lesson will increase 10%. So we multiply the per lesson costs by 110% or 1.1. The $5 processing fee will be a vertical translation of 5.

So the new costs can be found by

\[ f(x) = \begin{cases} 
(1.1)40x + 5 & \text{if } 0 < x \leq 4 \\
(1.1)30x + 5 & \text{if } 4 < x \leq 8 \text{ or } f(x) = \begin{cases} 
44x + 5 & \text{if } 0 < x \leq 4 \\
33x + 5 & \text{if } 4 < x \leq 8 .
\end{cases} \\
(1.1)25x + 5 & \text{if } x > 8 \\
27.5x + 5 & \text{if } x > 8
\end{cases} \]

PTS: 1 DIF: Average REF: 1718cbca-4683-11df-9c7d-001185f0d2ea
OBJ: 6-4.4 Problem-Solving Application NAT: NT.CCSS.MTH.10.9-12.F.BF.3
LOC: MTH.C.10.07.16.05.003 TOP: 6-4 Transforming Functions
MSC: DOK 3
22. ANS:

\[(f - g)(x) = 2x^2 + 13x - 10\]
\[(f - g)(x) = f(x) - g(x)\]
\[= (2x^2 + 8x - 4) - (-5x + 6)\]
\[= 2x^2 + 13x - 10\]

Substitute function rules.

Distribute the negative and combine like terms.

PTS: 1   DIF: Basic   REF: 171d6972-4683-11df-9c7d-001185f0d2ea
OBJ: 6-5.1 Adding and Subtracting Functions   NAT: NT.CCSS.MTH.10.9-12.F.BF.1.b
STA: CA.CACS.MTH.97.AL2.AII.24.0
LOC: MTH.C.10.07.15.02.001 | MTH.C.10.07.15.03.001   TOP: 6-5 Operations with Functions
MSC: DOK 2

23. ANS:

\[(fg)(x) = -8x^3 + 42x^2 + 46x - 60\]
\[(fg)(x) = f(x) \cdot g(x)\]
\[= (4x^2 + 3x - 5)(-2x + 12)\]
\[= 4x^2(-2x + 12) + 3x(-2x + 12) - 5(-2x + 12)\]
\[= -8x^3 + 48x^2 - 6x^2 + 36x - 10x - 60\]
\[= -8x^3 + 42x^2 + 46x - 60\]

Substitute function rules.

Distribute.

Multiply.

Combine like terms.

PTS: 1   DIF: Average   REF: 171f4bce-4683-11df-9c7d-001185f0d2ea
OBJ: 6-5.2 Multiplying and Dividing Functions   NAT: NT.CCSS.MTH.10.9-12.F.BF.1.c
STA: CA.CACS.MTH.97.AL2.AII.24.0
LOC: MTH.C.10.07.15.03.001   TOP: 6-5 Operations with Functions
MSC: DOK 2

24. ANS:

\[g(f(3)) = 111\]
\[g(f(3)) = g(3^3)\]
\[= g(27)\]
\[= 4(27) + 3\]
\[= 111\]

Simplify.

So, \(g(f(3)) = 111\).

PTS: 1   DIF: Average   REF: 171ff2de-4683-11df-9c7d-001185f0d2ea
OBJ: 6-5.3 Evaluating Composite Functions   NAT: NT.CCSS.MTH.10.9-12.F.BF.1.c
STA: CA.CACS.MTH.97.AL2.AII.24.0
LOC: MTH.C.10.07.15.05.006   TOP: 6-5 Operations with Functions
MSC: DOK 1
25. ANS:

\[ y = \pm \frac{4}{\sqrt[4]{x}} + 8; \]

The inverse is not a function. The domain is \([0, \infty)\) and the range is \((-\infty, \infty)\).

\[ y = (3x - 24)^4 \quad \text{Rewrite the function using } y \text{ instead of } f(x). \]

\[ x = (3y - 24)^4 \quad \text{Switch } x \text{ and } y \text{ in the equation.} \]

\[ \sqrt[4]{x} = \sqrt[4]{(3y - 24)^4} \quad \text{Take the fourth root of both sides.} \]

\[ \pm \sqrt[4]{x} = 3y - 24 \quad \text{Note the domain restriction } x \geq 0. \]

\[ y = \frac{1}{3} \left( \pm \sqrt[4]{x} + 24 \right) \quad \text{Isolate } y. \]

\[ y = \pm \frac{1}{3} \sqrt[4]{x} + 8 \quad \text{Simplify.} \]

Because of the \(\pm\) there are two \(y\)-values for all \(x > 0\). Thus, the inverse is not a function. The domain of the inverse is the range of \(f(x): [0, \infty)\). The range is the domain of \(f(x): (-\infty, \infty)\).
26. ANS:
Yes, \( f(g(x)) = g(f(x)) = x \).

Find the compositions \( f(g(x)) \) and \( g(f(x)) \).

\[
\begin{align*}
  f(g(x)) &= \frac{1}{3} \left( 5x - 20 \right) + 4 \\
  g(f(x)) &= 5 \left( \frac{1}{3} x + 4 \right) - 20 \\
  f(g(x)) &= (x - 4) + 4 \\
  g(f(x)) &= (x + 20) - 20 \\
  f(g(x)) &= x \\
  g(f(x)) &= x
\end{align*}
\]

Because \( f(g(x)) = g(f(x)) = x \), \( f \) and \( g \) are inverses.

Check The graphs are symmetric about the line \( y = x \).

27. ANS:

24 outfits

<table>
<thead>
<tr>
<th>Number of tops</th>
<th>times</th>
<th>Number of bottoms</th>
<th>times</th>
<th>Number of scarves</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td></td>
<td>2</td>
<td></td>
<td>3</td>
</tr>
</tbody>
</table>

The total number of outfits is \( 4 \times 2 \times 3 = 24 \).
28. ANS: 5,040 ways
Since the order matters, use the formula for permutations.

\[ 7! \cdot P_7 = \frac{7!}{(7-7)!} \]

Since 0! = 1, the number of ways is 7! = 5,040.

PTS: 1  DIF: Average  REF: 179001c2-4683-11df-9c7d-001185f0d2ea
OBJ: 7-1.2 Finding Permutations  NAT: NT.CCSS.MTH.10.9-12.S.CP.9
LOC: MTH.C.13.06.02.02.004  TOP: 7-1 Permutations and Combinations
KEY: permutation | ordering  MSC: DOK 3

29. ANS: 792 ways

**Step 1** Determine whether the problem represents a combination or a permutation.
The order does not matter because choosing a green shirt, a blue shirt, and a red shirt is the same as choosing a red shirt, a blue shirt, and a green shirt. It is a combination.

**Step 2** Use the formula for combinations.

The number of combinations of \( n \) items taken \( r \) at a time is \( \binom{n}{r} = \frac{n!}{r!(n-r)!} \).

\[
\binom{12}{5} = \frac{12!}{5!(12-5)!} = \frac{12!}{5!7!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot (7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)}
\]

Expand.

\[
\binom{12}{5} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \cdot \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{5 \cdot 4 \cdot 3 \cdot 2} \cdot \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}
\]

Divide out common factors.

\[
\binom{12}{5} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{12 \cdot 11 \cdot 10 \cdot 9}{5 \cdot 4 \cdot 3} = \frac{12 \cdot 11 \cdot 2 \cdot 3}{5 \cdot 4} = 72!
\]

Simplify.

There are 729 ways to select a group of 5 shirts from 12.

PTS: 1  DIF: Average  REF: 17923d0e-4683-11df-9c7d-001185f0d2ea
OBJ: 7-1.3 Application  NAT: NT.CCSS.MTH.10.9-12.S.CP.9
LOC: MTH.C.13.06.02.03.003  TOP: 7-1 Permutations and Combinations
MSC: DOK 3
30. **ANS:**

\[ \frac{1}{3} \]

There are six possible outcomes when a fair number cube is rolled. Because the number cube is fair, all outcomes are equally likely. There are two numbers greater than 4 on the number cube: 5 and 6. So the probability of rolling one of these numbers is \( \frac{2}{6} = \frac{1}{3} \).

**PTS:** 1  
**DIF:** Basic  
**REF:** 17949f6a-4683-11df-9c7d-001185f0d2ea  
**OBJ:** 7-2.1 Finding Theoretical Probability  
**TOP:** 7-2 Theoretical and Experimental Probability  
**KEY:** probability | theoretical probability  
**MSC:** DOK 2

31. **ANS:**

85.7%

\[ P(\text{different days}) = 1 - P(\text{Monday}) \]

Use the complement.

\[ = 1 - \left( \frac{1}{7} \right) \]

There are 7 days in the week.

\[ = 85.7\% \]

**PTS:** 1  
**DIF:** Basic  
**REF:** 1794c67a-4683-11df-9c7d-001185f0d2ea  
**OBJ:** 7-2.2 Application  
**TOP:** 7-2 Theoretical and Experimental Probability  
**MSC:** DOK 2

32. **ANS:**

0.05

Since order does not matter, use combinations.

The number of outcomes in the sample space is \( \binom{25}{6} = \frac{25!}{6!(25-6)!} = 177,100 \).

The number of favorable outcomes is \( \binom{17}{2} \binom{8}{4} = 9520 \).

The probability that in a random sample of 6 respondents from this survey, with exactly 2 favoring the proposed regulation and 4 opposing it, is

\[ \frac{\text{number of favorable outcomes}}{\text{number of outcomes in the sample space}} = \frac{9520}{177,100} \approx 0.05. \]

**PTS:** 1  
**DIF:** Average  
**REF:** 179701c6-4683-11df-9c7d-001185f0d2ea  
**OBJ:** 7-2.3 Finding Probability with Permutations or Combinations  
**NAT:** NT.CCSS.MTH.10.9-12.S.CP.9  
**STA:** CA.CACS.MTH.97.AL2.AII.19.0  
**LOC:** MTH.C.13.05.03.019 | MTH.C.13.06.02.03.003  
**TOP:** 7-2 Theoretical and Experimental Probability  
**MSC:** DOK 3
33. **ANS:**

21.5%

Find the ratio of the area of the region inside the square but outside the circle to the total area of the square.

The area of the region outside the circle is equal to the area of the square minus the area of the circle.

\[ A_o = A_s - A_c \]

\[ A_o = (4^2) - (\pi(2)^2) = 16 - 4\pi \]

\[
\frac{\text{Area outside region}}{\text{Area square}} = \frac{A_o}{A_s} = \frac{16 - 4\pi}{16} = 0.215 = 21.5\%
\]

**PTS:** 1  
**DIF:** Average  
**REF:** 17996422-4683-11df-9c7d-001185f0d2ea  
**OBJ:** 7-2.4 Finding Geometric Probability  
**LOC:** MTH.C.13.05.002  
**TOP:** 7-2 Theoretical and Experimental Probability  
**MSC:** DOK 3

34. **ANS:**

\[ \frac{12}{17} \]

When the spinner does not land on red, it must land on yellow or purple.

\[
\text{experimental probability} = \frac{\text{number of times the event occurs}}{\text{number of trials}} = \frac{13 + 11}{34} = \frac{24}{34} = \frac{12}{17}
\]

**PTS:** 1  
**DIF:** Basic  
**REF:** 17998b32-4683-11df-9c7d-001185f0d2ea  
**OBJ:** 7-2.5 Finding Experimental Probability  
**LOC:** MTH.C.13.05.02.001  
**TOP:** 7-2 Theoretical and Experimental Probability  
**MSC:** DOK 2

35. **ANS:**

The events are dependent because \( P(\text{Jessica green}) \) is different when Lila has already removed one ribbon from the bag.

\[ \frac{51}{380} \]

Before Lila drew a black ribbon from the bag \( P(\text{Jessica green}) = \frac{17}{20} \), after Lila drew it was \( \frac{17}{19} \).

The probability of drawing a black ribbon first is 3 out of 20, or \( \frac{3}{20} \).

For the second draw, there are 19 ribbons left, and there are still 17 green ribbons. The probability of drawing a green ribbon second is 17 out of 19, or \( \frac{17}{19} \).

To find the probability that both events happen, multiply the probabilities.

\[
P(\text{black and green}) = \frac{3}{20} \cdot \frac{17}{19} = \frac{51}{380}.
\]

**PTS:** 1  
**DIF:** Average  
**REF:** 179e28da-4683-11df-9c7d-001185f0d2ea  
**OBJ:** 7-3.2 Finding the Probability of Dependent Events  
**NAT:** NT.CCSS.MTH.10.9-12.S.CP.8 | NT.CCSS.MTH.10.9-12.S.CP.2  
**LOC:** MTH.C.13.05.03.009  
**TOP:** 7-3 Independent and Dependent Events  
**KEY:** events | independent events | outcomes | probability  
**MSC:** DOK 3
36. ANS: 0.121
   Use the Female row. Of 66,033,000 female labor force, 8,004,000 work in the Industry field.
   \[ P(\text{Industry | Female}) = \frac{8,004,000}{66,033,000} = 0.121 \]

37. ANS: independent; 0.16
   One outcome does not affect the other, so the events are independent.
   To find the probability that A and B both happen, multiply the probabilities.
   \[ P(A \text{ and } B) = P(A) \cdot P(B) = 0.8 \cdot 0.2 = 0.16 \]

38. ANS: 0.61

39. ANS: 0.23
40. ANS:
\[
\frac{1}{2}
\]

\[
P(5 \text{ or odd})
\]

= \(P(5) + P(\text{odd}) - P(5 \text{ and odd})\)

= \(\frac{1}{6} + \frac{3}{6} - \frac{1}{6}\)

5 is also an odd number.

= \(\frac{1}{2}\)

PTS: 1  DIF: Basic  REF: 17a54fee-4683-11df-9c7d-001185f0d2ea
OBJ: 7-5.2 Finding Probabilities of Inclusive Events  NAT: NT.CCSS.MTH.10.9-12.S.CP.7
LOC: MTH.C.13.05.03.013  TOP: 7-5 Compound Events
MSC: DOK 2

41. ANS:
6, 11, 21, 41, 81

The first term is given \(a_1 = 6\).

Substitute this value into the rule to find the next term.

Continue using each term to find the next term.

<table>
<thead>
<tr>
<th>(n)</th>
<th>(2a_{n-1} - 1)</th>
<th>(a_n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>2(6) - 1</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>2(11) - 1</td>
<td>21</td>
</tr>
<tr>
<td>4</td>
<td>2(21) - 1</td>
<td>41</td>
</tr>
<tr>
<td>5</td>
<td>2(41) - 1</td>
<td>81</td>
</tr>
</tbody>
</table>

PTS: 1  DIF: Average  REF: 17f19b6e-4683-11df-9c7d-001185f0d2ea
OBJ: 9-1.1 Finding Terms of a Sequence by Using a Recursive Formula
NAT: NT.CCSS.MTH.10.9-12.F.BF.2  LOC: MTH.C.13.06.01.01.003
TOP: 9-1 Introduction to Sequences  MSC: DOK 2
42. ANS:

-3, -1, 3, 11, 27

Make a table. Evaluate the sequence for \( n = 1 \) through \( n = 5 \).

<table>
<thead>
<tr>
<th>( n )</th>
<th>( 2^n - 5 )</th>
<th>( a_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2(^1) - 5</td>
<td>-3</td>
</tr>
<tr>
<td>2</td>
<td>2(^2) - 5</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>2(^3) - 5</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>2(^4) - 5</td>
<td>11</td>
</tr>
<tr>
<td>5</td>
<td>2(^5) - 5</td>
<td>27</td>
</tr>
</tbody>
</table>

The first 5 terms are -3, -1, 3, 11, and 27.

43. ANS:

\( a_n = 26 - 2.9n \)

Examine the differences.

23.1 20.2 17.3 14.4 11.5 8.6
First difference 2.9 2.9 2.9 2.9 2.9

The first differences are constant, so the sequence is linear.

\( a_n = 23.1 - 2.9(n - 1) \)  \( a_n = 23.1 - 2.9(n - 1) \)  \( a_n = 23.1 - 2.9n + 2.9 \)  \( a_n = 26 - 2.9n \)

Distribute and simplify.

PTS: 1  DIF: Average  REF: 17f3fdca-4683-11df-9c7d-001185f0d2ea
OBJ: 9-1.3 Writing Rules for Sequences  LOC: MTH.C.13.06.03.01.003
TOP: 9-1 Introduction to Sequences  MSC: DOK 3
44. ANS:

The graph appears exponential and reaches an area of 4.5 thousand square yards after 6 years.

Because the island has an area of 10.2 thousand square yards and then decreases to 85% of its previous area, the recursive rule is \( a_1 = 10.2 \) and \( a_n = 0.85a_{n-1} \).

Use this rule to find other terms of the sequence.

\[ a_2 = 0.85a_1 = 0.85(10.2) = 8.67 \]
\[ a_3 = 0.85a_2 = 0.85(8.67) = 7.37 \]

The graph appears to be exponential.

Use the pattern to write the explicit rule.

\[ a_n = (10.2)0.85^{n-1} \]

Use this rule to find the area after 6 years.

\[ a_6 = (10.2)0.85^5 = 4.5 \]

The island has approximately 4.5 thousand square yards of area after 6 years.
45. ANS:
36 square units

**Step 1** Find the length of the sides of the third square.
The sides of the first square are 3 units long. The sides of the second square are $3 \sqrt{2}$ units long. The sides of the third square are $3 \sqrt{2} \cdot \sqrt{2} = 6$ units long.

**Step 2** Find the area of the third square.
$A = 6 \times 6 = 36$ square units

PTS: 1  DIF: Advanced  REF: 17f8c282-4683-11df-9c7d-001185f0d2ea
LOC: MTH.C.13.06.01.02.003  TOP: 9-1 Introduction to Sequences
MSC: DOK 2

46. ANS:
\[ \sum_{k=1}^{6} (-1)^k \left( \frac{1}{2k} \right) \]

Find a rule for the $k$th term.
\[ a_k = (-1)^k \left( \frac{1}{2k} \right) \quad \text{Explicit formula} \]

Write the notation for the first 6 items.
\[ \sum_{k=1}^{6} (-1)^k \left( \frac{1}{2k} \right) \quad \text{Summation notation} \]

PTS: 1  DIF: Average  REF: 17fb24de-4683-11df-9c7d-001185f0d2ea
OBJ: 9-2.1 Using Summation Notation  STA: CA.CACS.MTH.97.AL2.AI2.22.0
LOC: MTH.C.13.06.01.02.003  TOP: 9-2 Series and Summation Notation
MSC: DOK 3
47. **ANS:**
   6
   Expand the series by replacing $k$. Then evaluate the sum.
   \[
   \sum_{k=2}^{6} (-1)^k (7k - k^2)
   \]
   \[
   = (-1)^2 ((7)(2) - 2^2) + (-1)^3 ((7)(3) - 3^2)
   \]
   \[
   + (-1)^4 ((7)(4) - 4^2) + (-1)^5 ((7)(5) - 5^2) + (-1)^6 ((7)(6) - 6^2)
   \]
   \[
   = 10 - 12 + 12 - 10 + 6
   \]
   \[
   = 6
   \]

   **PTS:** 1  **DIF:** Average  **REF:** 17fb4bee-4683-11df-9c7d-001185f0d2ea
   **OBJ:** 9-2.2 Evaluating a Series  **STA:** CA.CACS.MTH.97.AL2.AII.22.0
   **LOC:** MTH.C.13.06.01.02.004  **TOP:** 9-2 Series and Summation Notation
   **MSC:** DOK 2

48. **ANS:**
   253
   \[
   \sum_{k=1}^{22} k = \frac{n(n+1)}{2} = \frac{22(23)}{2} = 253
   \]
   Use the summation formula for a linear series.

   **PTS:** 1  **DIF:** Average  **REF:** 17fd873a-4683-11df-9c7d-001185f0d2ea
   **OBJ:** 9-2.3 Using Summation Formulas  **STA:** CA.CACS.MTH.97.AL2.AII.22.0
   **LOC:** MTH.C.13.06.01.02.004  **TOP:** 9-2 Series and Summation Notation
   **MSC:** DOK 2

49. **ANS:**
   Yes; common difference 8; next 3 terms are 39, 47, 55
   For a sequence to be an arithmetic sequence, each number subtracted from the one before it should result
   in a common difference.
   This sequence is arithmetic. Each term differs from the previous one by 8.

   **PTS:** 1  **DIF:** Basic  **REF:** 180010a6-4683-11df-9c7d-001185f0d2ea
   **OBJ:** 9-3.1 Identifying Arithmetic Sequences
   **LOC:** MTH.C.13.06.01.01.001 | MTH.C.13.06.01.01.002
   **TOP:** 9-3 Arithmetic Sequences and Series  **KEY:** arithmetic sequence
   **MSC:** DOK 2
Find a specific term from a given sequence by using the equation \( a_n = a_1 + (n - 1)d \), where:

- \( a_n \) = your result
- \( a_1 \) = the initial term of the sequence
- \( n \) = the number in the sequence you want to calculate
- \( d \) = the common difference between the terms

\( n \) is given in the problem, \( a_1 \) is the first term in the sequence, and \( d \) is the difference between adjacent terms.

**Step 1** Find the common difference.
\[
a_{13} = a_7 + (13 - 7)d
\]
\[
55 = 25 + (6)d
\]
\[
5 = d
\]

**Step 2** Find \( a_1 \).
\[
a_{12} = a_1 + (7 - 1)d
\]
\[
25 = a_1 + (6)5
\]
\[
-5 = a_1
\]

**Step 3** Write a rule for the sequence and evaluate to find \( a_5 \).
\[
a_n = a_1 + (n - 1)d
\]
\[
a_5 = -5 + (5 - 1)5
\]
\[
a_5 = 15
\]
52. ANS:

1313

Find the 1st and 13th terms.

\[ a_1 = 15(1) - 4 = 11 \]

\[ a_{13} = 15(13) - 4 = 191 \]

Find \( S_{13} \):

\[
S_n = n \left( \frac{a_1 + a_n}{2} \right)
\]

\[
S_{13} = 13 \left( \frac{a_1 + a_{13}}{2} \right)
\]

\[
S_{13} = 13 \left( \frac{11 + 191}{2} \right)
\]

\[ S_{13} = 1313 \]

PTS: 1  DIF: Average  REF: 180710aa-4683-11df-9c7d-001185f0d2ea

OBJ: 9-3.5 Finding the Sum of an Arithmetic Series  STA: CA.CACS.MTH.97.AL2.AII.22.0

LOC: MTH.C.13.06.01.02.006  TOP: 9-3 Arithmetic Sequences and Series

MSC: DOK 2

53. ANS:

\[
\sum_{k=1}^{7} (9 - 4k)
\]

The general form for the \( k \)th term in an arithmetic series is \( a_k = a_0 + d(k - 1) \) where \( a_0 \) is the first term \( d \) is the common difference between consecutive terms. In the question, \( a_0 = 5 \) and \( d = -4 \). Therefore,

\[
a_k = a_0 + d(k - 1)
\]

\[
= 5 - 4(k - 1)
\]

\[
= 9 - 4k
\]

The general form for a partial arithmetic sum is \( \sum_{k=1}^{n} a_k \) where \( n \) is the total number of terms in the series.

Therefore,

\[
\sum_{k=1}^{7} a_k = \sum_{k=1}^{7} (9 - 4k)
\]

PTS: 1  DIF: Advanced  REF: 18097306-4683-11df-9c7d-001185f0d2ea

STA: CA.CACS.MTH.97.AL2.AII.22.0  LOC: MTH.C.13.06.01.02.003

TOP: 9-3 Arithmetic Sequences and Series  MSC: DOK 3
Step 1 Find the common ratio.
\[ r = \frac{a_2}{a_1} = \frac{12}{-4} = -3 \]

Step 2 Write a rule, and evaluate for \( n = 7 \).
\[ a_n = a_1 r^{n-1} \] General rule
\[ a_7 = -4(-3)^{7-1} \] Substitute -4 for \( a_1 \), 7 for \( n \), and -3 for \( r \).
\[ a_7 = -2,916 \]

The 7th term is -2,916.
55. ANS: 256

**Step 1** Find the common ratio.

\[ a_5 = a_3 r^{(5-3)} \]

Use the given terms.

\[ a_5 = a_3 r^2 \]

Simplify.

\[ 64 = 16r^2 \]

Substitute 64 for \( a_5 \) and 16 for \( a_3 \).

\[ 4 = r^2 \]

Divide both sides by 16.

\[ \pm 2 = r \]

Take the square root of both sides.

**Step 2** Find \( a_1 \).

Consider both the positive and negative values for \( r \).

\[
\begin{align*}
16 &= a_1 (2)^{3-1} \\
4 &= a_1
\end{align*}
\]

\[
\begin{align*}
16 &= a_1 (-2)^{3-1} \\
4 &= a_1
\end{align*}
\]

Use \( a_3 = 16 \) and \( r = \pm 2 \).

Simplify.

**Step 3** Write the rule and evaluate for \( a_7 \).

Consider both the positive and negative values for \( r \).

\[
\begin{align*}
a_n &= a_1 r^{n-1} \\
a_7 &= 4(2)^{7-1} \\
a_7 &= 256
\end{align*}
\]

\[
\begin{align*}
a_n &= a_1 r^{n-1} \\
a_7 &= 4(-2)^{7-1} \\
a_7 &= 256
\end{align*}
\]

Substitute for \( a_1 \) and \( r \).

Evaluate for \( n = 7 \).

Simplify.

The 7th term is 256.
56.  

ANS:  
6.666666  

Step 1 Find the common ratio.  
\[ r = \frac{+0.6}{6} = 0.1 \]

Step 2 Find \( S_8 \) with \( a_1 = 6, r = 0.1, \) and \( n = 8. \)  

\[ S_n = a_1 \left( \frac{1-r^n}{1-r} \right) \quad \text{Sum formula} \]

\[ S_8 = 6 \left( \frac{1-(0.1)^8}{1-0.1} \right) \quad \text{Substitute.} \]

\[ S_8 = 6 \left( \frac{1-0.00000001}{1-0.1} \right) \quad \text{Use the order of operations. Calculate exponents before adding or subtracting.} \]

\[ S_8 = 6 \left( \frac{0.99999999}{0.9} \right) = \quad \text{Simplify.} \]

6.666666 

The sum of the first 8 terms of the geometric sequence is 6.666666.

PTS: 1  DIF: Average  REF: 1810c12a-4683-11df-9c7d-001185f0d2ea  
OBJ: 9-4.5 Finding the Sum of a Geometric Series  NAT: NT.CCSS.MTH.10.9-12.A.SSE.4  
STA: CA.CACS.MTH.97.AL2.AII.22.0  LOC: MTH.C.13.06.01.02.008  
TOP: 9-4 Geometric Sequences and Series  MSC: DOK 2
57. **ANS:**
4, −8, 16

**Step 1** Find the common ratio.

\[ a_{11} = a_6 r^{11-6} \]

Use the given terms.

\[ a_{11} = a_6 r^5 \]

Simplify.

\[ 4,096 = -128 r^5 \]

Substitute 4,096 for \( a_{11} \) and −128 for \( a_6 \).

\[ -32 = r^5 \]

Divide both sides by −128.

\[ -2 = r \]

Take the fifth root of both sides.

**Step 2** Find \( a_1 \).

\[ a_6 = a_1 r^{6-1} \]

Substitute \( a_6 = -128 \) and \( r = -2 \). Simplify.

\[ -128 = a_1 (-2)^5 \]

\[ 4 = a_1 \]

Divide both sides by \((-2)^5\).

**Step 3** Find \( a_2 \) and \( a_3 \).

\[ a_n = a_1 r^{n-1} \]

General rule for geometric sequence

\[ a_2 = a_1 (-2)^{2-1} \]

Substitute \( n = 2, a_1 = 4, \) and \( r = -2 \).

\[ a_2 = 4(-2)^1 = -8 \]

Simplify.

\[ a_3 = a_1 (-2)^{3-1} \]

Substitute \( n = 3, a_1 = 4, \) and \( r = -2 \).

\[ a_3 = 4(-2)^2 = 16 \]

Simplify.
59. ANS:  
56,713 ft  
\[ \tan \theta = \frac{\text{opp.}}{\text{adj.}} \]
\[ \tan 10^\circ = \frac{10,000}{x} \]
Substitute 10° for \( \theta \), 10,000 for opp., and \( x \) for adj.
\[ x(\tan 10^\circ) = 10,000 \]
Multiply both sides by \( x \).
\[ x = \frac{10,000}{\tan 10^\circ} = 56,713 \]
Divide both sides by \( \tan 10^\circ \). Use a calculator to simplify.

The land distance from the airplane to the airport is about 56,713 feet.
60. **ANS:**

41 ft

**Step 1** Draw and label a diagram to represent the information given in the problem.

**Step 2** Let \( x \) represent the height of the building from the surveyor’s eye level. Determine the value of \( x \).

\[
\tan \theta = \frac{\text{opposite}}{\text{adjacent}}
\]

Use the tangent function.

\[
\tan 41.7 = \frac{x}{40}
\]

Substitute using \( x \) for opposite, 41.7 for \( \theta \), and 40 for adjacent.

\[
40(\tan 41.7) = x
\]

Multiply both sides by 40.

\[
x \approx 35.6387
\]

Use a calculator to solve for \( x \).

**Step 3** Determine the overall height of the building.

\[
\text{height} = x + 5 = 36 + 5
\]

The surveyor’s eye level is 5 ft above the ground, so add 5 ft to the overall height of the building.

\[
\text{height} = 41
\]

The height of the building is about 41 ft.
61. ANS:
\[
\sin \theta = \frac{7}{25}; \quad \cos \theta = \frac{24}{25};
\]
\[
\tan \theta = \frac{7}{24}; \quad \csc \theta = \frac{25}{7};
\]
\[
\sec \theta = \frac{25}{24}; \quad \cot \theta = \frac{24}{7}.
\]

**Step 1** Find the length of the hypotenuse.

\[
a^2 + b^2 = c^2 \quad \text{Pythagorean Theorem}
\]
\[
c^2 = (14)^2 + (48)^2 \quad \text{Substitute 14 for } a \text{ and 48 for } b.
\]
\[
c^2 = 2,500 \quad \text{Simplify.}
\]
\[
c = 50 \quad \text{Solve for } c. \text{ Eliminate the negative solution.}
\]

**Step 2** Find the function values. Simplify.

\[
\sin \theta = \frac{\text{opp.}}{\hyp} = \frac{14}{50} = \frac{7}{25}.
\]
\[
\cos \theta = \frac{\text{adj.}}{\hyp} = \frac{48}{50} = \frac{24}{25}.
\]
\[
\tan \theta = \frac{\text{opp.}}{\text{adj.}} = \frac{14}{48} = \frac{7}{24}.
\]
\[
\csc \theta = \frac{1}{\sin \theta} = \frac{50}{14} = \frac{25}{7}.
\]
\[
\sec \theta = \frac{1}{\cos \theta} = \frac{50}{48} = \frac{25}{24}.
\]
\[
\cot \theta = \frac{\text{adj.}}{\text{opp.}} = \frac{48}{14} = \frac{24}{7}.
\]

PTS: 1   DIF: Basic   REF: 1849f9ca-4683-11df-9c7d-001185f0d2ea
OBJ: 10-1.5 Finding All Trigonometric Ratios
LOC: MTH.C.14.02.02.002 | MTH.C.14.02.02.004 | MTH.C.14.02.02.006
TOP: 10-1 Right-Angle Trigonometry   MSC: DOK 2

62. ANS:
\[
306^\circ = \left(\frac{17\pi}{10}\right)\text{ radians} \times \left(\frac{180^\circ}{\pi\text{ radians}}\right) = 306^\circ \quad \text{Multiply by } \left(\frac{180^\circ}{\pi\text{ radians}}\right).
\]

PTS: 1   DIF: Basic   REF: 1853833a-4683-11df-9c7d-001185f0d2ea
OBJ: 10-3.1 Converting Between Degrees and Radians
LOC: MTH.C.14.01.003
TOP: 10-3 The Unit Circle   MSC: DOK 2
63. \( \text{ANS:} \)
\[
-\frac{7\pi}{36}
\]
Multiply \(-35^\circ\) by \(\frac{\pi \text{ radians}}{180^\circ}\).

PTS: 1  DIF: Average  REF: 1855be86-4683-11df-9c7d-001185f0d2ea
OBJ: 10-3.1 Converting Between Degrees and Radians  LOC: MTH.C.14.01.002
TOP: 10-3 The Unit Circle  KEY: convert | degrees | radians
MSC: DOK 2

64. \( \text{ANS:} \)
\[
\frac{\sqrt{3}}{2}
\]
The angle passes through the point \(\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)\) on the unit circle.

\[
\cos \theta = x
\]
\[
\cos 30^\circ = \frac{\sqrt{3}}{2}
\]

PTS: 1  DIF: Average  REF: 185820e2-4683-11df-9c7d-001185f0d2ea
OBJ: 10-3.2 Using the Unit Circle to Evaluate Trigonometric Functions
LOC: MTH.C.14.03.02.015  TOP: 10-3 The Unit Circle
MSC: DOK 2
65. **ANS:**
   14 miles

**Step 1** Find the radius of the tire.

\[ r = \frac{d}{2} = 13 \text{ inches}. \]

**Step 2** Find the angle \( \theta \) (in radians) through which the point on the tire rotates in 1 hour.

\[
\begin{align*}
3 \text{ rotations} \cdot \frac{2\pi \text{ radians}}{1 \text{ rotation}} = \frac{6\pi \text{ radians}}{1 \text{ second}}
\end{align*}
\]

Use proportions to convert seconds to hours.

\[
\begin{align*}
\frac{6\pi \text{ radians}}{1 \text{ second}} \cdot \frac{60 \text{ seconds}}{1 \text{ minute}} \cdot \frac{60 \text{ minutes}}{1 \text{ hour}} = \frac{21600\pi \text{ radians}}{1 \text{ hour}}
\end{align*}
\]

**Step 3** Find the length of the arc intercepted by \( 21,600\pi \text{ radians} \).

\[ s = r\theta = 13(21,600\pi) = 280,800\pi \text{ inches} \]

Use proportions to convert inches to miles.

\[
\begin{align*}
\frac{280,800\pi \text{ inches}}{1} \cdot \frac{1 \text{ foot}}{12 \text{ inches}} \cdot \frac{1 \text{ mile}}{5,280 \text{ feet}} = 4.432\pi \text{ miles} \approx 14 \text{ miles}
\end{align*}
\]

PTS: 1 DIF: Average REF: 185a833e-4683-11df-9c7d-001185f0d2ea
OBJ: 10-3.4 Application TOP: 10-3 The Unit Circle
MSC: DOK 2

66. **ANS:**
   37°

Find the value of \( \theta \).

\[
\tan \theta = \frac{\text{opp.}}{\text{adj.}}
\]

Use the tangent ratio.

\[
\tan \theta = \frac{35}{46}
\]

Substitute 35 for opp. and 46 for adj. Then simplify.

\[
\theta = \tan^{-1} \left( \frac{35}{46} \right) = 37°
\]

Use the inverse tangent function on your calculator to find the value of \( \theta \).

The angle of elevation of the sun from the end of the shadow is 37°.

PTS: 1 DIF: Basic REF: 185f6f06-4683-11df-9c7d-001185f0d2ea
OBJ: 10-4.3 Application LOC: MTH.C.14.04.03.002
TOP: 10-4 Inverses of Trigonometric Functions MSC: DOK 2
67. ANS:

[1.] $\theta \approx 62^\circ$

[2.] $h \approx 482$ ft

[1.] The angle of elevation, $\theta$, may be found using the tangent function, $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$. The opposite side is the distance from eye level to the top of the pyramid, or 495 feet. The adjacent side is the distance from the archaeologist to the base of the pyramid, or 260 ft.

$$\tan \theta = \frac{495}{260}$$

$$\theta = \tan^{-1} \frac{495}{260} \approx 62^\circ$$

[2.] The height of the pyramid, $h$, from the archaeologist’s eye-level to the top, may be found using the tangent function, $\tan \theta = \frac{h}{\text{adjacent}}$. The angle of elevation is 50°. The adjacent side is the distance from the archaeologist to the base of the pyramid, or 400 ft.

$$\tan 50^\circ = \frac{h}{400}$$

$$h = 400 \tan 50^\circ \approx 477$$ ft

Adding 5 ft to this gives a total height of 482 ft.

PTS: 1 DIF: Advanced REF: 18640cae-4683-11df-9e7d-001185f0d2ea
NAT: NT.CCSS.MTH.10.9-12.F.TF.7 LOC: MTH.C.14.04.03.002
TOP: 10-4 Inverses of Trigonometric Functions MSC: DOK 3
Step 1 Find the third angle measure.
\[ m\angle M + m\angle N + m\angle P = 180^\circ \]
\[ m\angle M + 118^\circ + 33^\circ = 180^\circ \]
\[ m\angle M = 29^\circ \]

Step 2 Find the unknown side lengths.
\[
\frac{\sin M}{m} = \frac{\sin N}{n} \quad \text{Law of Sines} \quad \frac{\sin M}{m} = \frac{\sin P}{p}
\]
\[
\frac{\sin 29^\circ}{15} = \frac{\sin 118^\circ}{n} \quad \text{Substitute.} \quad \frac{\sin 29^\circ}{15} = \frac{\sin 33^\circ}{p}
\]
\[ n = \frac{15 \sin 118^\circ}{\sin 29^\circ} \quad \text{Solve for the unknown side.} \quad p = \frac{15 \sin 33^\circ}{\sin 29^\circ}
\]
\[ n \approx 27.3 \quad p \approx 16.9 \]
69. ANS:
Two triangles are possible.
c_1 = 162.3 m; c_2 = 34.6 m;
m\angle B_1 = 60.6^\circ; m\angle B_2 = 119.4^\circ
m\angle C_1 = 70.4^\circ; m\angle C_2 = 11.6^\circ

Step 1 Determine the number of triangles.
Let \( h \) be the altitude from \( \angle C \) to side \( c \). Then, \( b \) is the hypotenuse of the newly created triangle.

\[
\sin 49^\circ = \frac{\text{opp}}{\text{adj}} = \frac{h}{150}
\]
In this case, \( m\angle A \) is acute. Find \( h \).

\[
h = 150(\sin 49^\circ) = 113
\]

\[
113 < 130 < 150
\]

\( h < a < b \)
Because \( h < a < b \), there are two possible triangles.

Step 2 Find \( m\angle B_1 \) and \( m\angle B_2 \).

\[
\frac{\sin A}{a} = \frac{\sin B}{b} \quad \text{Law of Sines}
\]

\[
\sin 49^\circ = \frac{\sin B}{150} \quad \text{Solve for } B.
\]

\[
\sin B_1 = 150 \left( \frac{\sin 49^\circ}{\sin 130^\circ} \right)
\]
Let \( m\angle B_1 \) be the acute angle.

\[
\sin B_1 = 0.870819
\]

\[
m\angle B_1 = 60.6^\circ
\]
Use the inverse sine on your calculator.

\[
m\angle B_2 = 180^\circ - 60.6^\circ = 119.4^\circ
\]
The reference angle of \( \angle B_2 \) is 60.6°.

Step 3 Find the other unknown measures of the two triangles.

Solve for \( m\angle C_1 \)

\[
m\angle A_1 + m\angle B_1 + m\angle C_1 = 180^\circ
\]

\[
m\angle A_1 + m\angle B_1 + m\angle C_1 = 180^\circ
\]

\[
m\angle C_1 = 180^\circ - (60.6^\circ + 49^\circ)
\]

\[
m\angle C_1 = 70.4^\circ
\]

\[
m\angle C_1 = 11.6^\circ
\]

Solve for \( c_1 \).

\[
\frac{\sin A}{a} = \frac{\sin C_1}{c_1} \quad \text{Law of Sines}
\]

\[
\sin 49^\circ = \frac{\sin 70.4^\circ}{c_1}
\]
Substitute.

\[
\sin 49^\circ = \frac{130}{c_1}
\]
Solve.

\[
c_1 = \frac{130 \sin 70.4^\circ}{\sin 49^\circ}
\]
\[
c_1 = 162.3
\]

Solve for \( c_2 \).

\[
\frac{\sin A}{a} = \frac{\sin C_2}{c_2}
\]

\[
\sin 49^\circ = \frac{\sin 11.6^\circ}{c_2}
\]
Substitute.

\[
\sin 49^\circ = \frac{130 \sin 11.6^\circ}{c_2}
\]
Solve.

\[
c_2 = \frac{130 \sin 11.6^\circ}{\sin 49^\circ}
\]
\[
c_2 = 34.6
\]
PTS: 1  DIF: Average  REF: 1868d166-4683-11df-9c7d-001185f0d2ea
OBJ: 10-5.3 Application
LOC: MTH.C.14.06.01.001  TOP: 10-5 The Law of Sines
MSC: DOK 3
Using the Law of Cosines, the distance is about 3.0 miles.

**Step 1** Find \( p \).

Find the angle \( AB \) makes with a line parallel to the x-axis.

![Diagram](image)

Note that \( FB \parallel AG \) so \( m \angle GAB = m \angle FBA \) by the Alternate Interior Angle Theorem.

\[
\tan p = m = \frac{3 - 2}{5 - 1} = \frac{1}{4}
\]

\[
p = \tan^{-1} \left( \frac{1}{4} \right) \approx 14^\circ
\]

**Step 2** Find \( m \angle BAC \) and \( m \angle ABC \).

![Diagram](image)

From the problem, \( m \angle FAC = 60^\circ \) and \( m \angle GBC = 40^\circ \).

By the Angle Addition Postulate,

\[
m \angle ABC = m \angle ABG + m \angle GBC
\]

\[
60^\circ = 14^\circ + m \angle BAC
\]

\[
m \angle ABC = 14^\circ + 40^\circ
\]

\[
m \angle BAC = 46^\circ
\]

\[
m \angle ABC = 54^\circ
\]

**Step 3** Solve for the necessary parts of \( \triangle ABC \).

Create a triangle, labeling the tree as point \( C \), \( m \angle A = 46^\circ \) and \( m \angle B = 54^\circ \).

![Diagram](image)

Using the distance formula, \( AB = \sqrt{(5 - 1)^2 + (3 - 2)^2} = 4.1 \) mi.

From the Sum Triangle Theorem, \( m \angle C = 180 - 46 - 54 = 80^\circ \).

Using the Law of Sines, find \( BC \).

\[
\frac{BC}{\sin A} = \frac{AB}{\sin C}
\]

\[
\frac{BC}{\sin 46^\circ} = \frac{4.1}{\sin 80^\circ}
\]

\[
BC = \sin 46^\circ \cdot \frac{4.1}{\sin 80^\circ}
\]
\[ BC = 3.0 \text{ mi.} \]

**Step 1** Solve for \( y \) so the expression can be used in a graphing calculator.

\[
4y^2 = 64 - 16x^2
\]

Subtract \( 16x^2 \) from both sides.

\[
y^2 = \frac{64 - 16x^2}{4}
\]

Divide both sides by 4.

\[
y = \pm \sqrt{\frac{64 - 16x^2}{4}}
\]

Take the square root of both sides.

**Step 2** Use two equations to see the complete graph.

\[
y = \sqrt{\frac{64 - 16x^2}{4}} \quad \text{and} \quad y = -\sqrt{\frac{64 - 16x^2}{4}}
\]

The graphs meet to form a complete ellipse, even though it may not appear that way on a graphing calculator.

The graph is an ellipse with center (0, 0) and intercepts (2, 0), (−2, 0), (0, 4), and (0, −4).
72. ANS:

The graph is a hyperbola that opens vertically with vertices at (0,9) and (0,−9).

**Step 1** Solve for $y$.

\[ y^2 = x^2 + 81 \]

\[ y = \pm \sqrt{x^2 + 81} \]

**Step 2** Use two equations to see the complete graph.

\[ y_1 = \sqrt{x^2 + 81} \text{ and } y_2 = -\sqrt{x^2 + 81} \]

PTS: 1  DIF: Average  REF: 173e8f2e-4683-11df-9c7d-001185f0d2ea
OBJ: 12-1.2 Graphing Parabolas and Hyperbolas on a Calculator
STA: CA.CACS.MTH.97.AL2.AII.17.0
LOC: MTH.C.10.09.04.003 | MTH.C.10.09.04.010
TOP: 12-1 Introduction to Conic Sections  MSC: DOK 3

73. ANS:

center \((-5, -3)\); radius 5

**Step 1** Use the midpoint formula to find the center.

\[ \left( \frac{-9 + -1}{2}, \frac{-6 + 0}{2} \right) = (-5, -3) \]

The center is \((-5, -3)\).

**Step 2** Use the Distance Formula with \((-5, -3)\) and \((-9, -6)\) to find the radius.

\[ r = \sqrt{(-5 - (-9))^2 + (-3 - (-6))^2} = 5 \]

The radius is 5.

PTS: 1  DIF: Basic  REF: 173eca7a-4683-11df-9c7d-001185f0d2ea
OBJ: 12-1.3 Finding the Center and Radius of a Circle
LOC: MTH.C.10.09.04.02.005 | MTH.C.11.05.04.003 | MTH.C.11.05.04.008
TOP: 12-1 Introduction to Conic Sections  MSC: DOK 2
74. **ANS:**

$$36 = (x - 8)^2 + (y - 7)^2$$

Use the Distance Formula with \((x_2, y_2) = (x, y)\), \((x_1, y_1) = (8, 7)\), and distance equal to the radius, 6.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$ Use the Distance Formula.

$$6 = \sqrt{(x - 8)^2 + (y - 7)^2}$$ Substitute.

$$6^2 = (x - 8)^2 + (y - 7)^2$$ Square both sides.

$$36 = (x - 8)^2 + (y - 7)^2$$

75. **ANS:**

$$\frac{y^2}{100} + \frac{x^2}{64} = 1$$

**Step 1** Choose the appropriate form of the equation.

$$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1.$$ Because the vertical axis is longer.

**Step 2** Identify the values of \(a\) and \(c\).

\(a = 10;\) The vertex \((0, -10)\) gives the value of \(a\).

\(c = 6;\) The focus \((0, 6)\) gives the value of \(c\).

**Step 3** Use the equation \(c^2 = a^2 - b^2\) to find the value of \(b^2\).

$$6^2 = 10^2 - b^2$$

$$64 = b^2$$

**Step 4** Write the equation

$$\frac{y^2}{100} + \frac{x^2}{64} = 1$$
Step 1 Rewrite the equation as \( \frac{(x-6)^2}{10^2} + \frac{(y+5)^2}{8^2} = 1 \)

Step 2 Identify the values of \( h, k, a, \) and \( b \).

\( h = 6 \) and \( k = -5 \), so the center is \((6, -5)\).
\( a = 10 \) and \( b = 8 \); because \( 10 > 8 \) the major axis is horizontal.

Step 3 The vertices are \((6 \pm 10, -5)\), or \((16, -5)\) and \((-4, -5)\), and the co-vertices are \((6, -5 \pm 8)\), or \((6, 3)\) and \((6, -13)\).

\[ \frac{x^2}{8000^2} + \frac{y^2}{6000^2} = 1 \]

Equation of an ellipse, with a horizontal major axis

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]

Substitute 8000 for \( a \) and 6000 for \( b \).
The vertex and focus are on the vertical axis so the equation will be of the form:

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1.$$

Step 2 The vertex is (0, 6) and the focus is (0, 8), so $a = 6$ and $c = 8$. Use $c^2 = a^2 + b^2$ to solve for $b^2$.

$$28 = 6^2 + b^2$$

Step 3 The equation of the hyperbola is $$\frac{y^2}{36} - \frac{x^2}{28} = 1.$$
79. **ANS:**

   Vertices: \((-2, 6)\) and \((-2, -4)\), Co-vertices: \((1, 1)\) and \((-5, 1)\)

   Asymptotes: \(y - 1 = \frac{5}{3}(x + 2)\) and \(y - 1 = -\frac{5}{3}(x + 2)\)

**Step 1** The equation is of the form \(\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1\), so the transverse axis is vertical with center \((-2, 1)\).

**Step 2** Because \(a = 5\) and \(b = 3\), the vertices are \((-2, 6)\) and \((-2, -4)\) and the co-vertices are \((1, 1)\) and \((-5, 1)\).

**Step 3** The equations of the asymptotes are \(y - 1 = \frac{5}{3}(x + 2)\) and \(y - 1 = -\frac{5}{3}(x + 2)\).

**Step 4** Draw a box using the vertices and co-vertices. Draw the asymptotes through the corners of the box.

**Step 5** Draw the hyperbola using the vertices and the asymptotes.
80. ANS: 
\[ y = \frac{1}{56}x^2 \]

**Step 1** Because the directrix is a horizontal line, the equation is in the form \( y = \frac{1}{4p}x^2 \). The vertex is above the directrix, so the graph will open upward.

**Step 2** Because the directrix is \( y = -14 \), \( p = 14 \) and \( 4p = 56 \).

**Step 3** The equation of the parabola is \( y = \frac{1}{56}x^2 \).

PTS: 1 DIF: Average REF: 1759046e-4683-11df-9c7d-001185f0d2ea
OBJ: 12-5.2 Writing Equations of Parabolas NAT: NT.CCSS.MTH.10.9-12.G.GPE.2
LOC: MTH.C.10.09.04.009 TOP: 12-5 Parabolas
MSC: DOK 2

81. ANS:

ellipse

The standard form of an ellipse, where \( a > b \), can be written as

\[
\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1, \text{ where the major axis is horizontal, or}
\]

\[
\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1, \text{ where the major axis is vertical.}
\]

\[
\frac{(x-2)^2}{3^2} + \frac{(y-4)^2}{7^2} = 1 \text{ is an ellipse with a vertical axis.}
\]

PTS: 1 DIF: Advanced REF: 175b8dda-4683-11df-9c7d-001185f0d2ea
OBJ: 12-6.1 Identifying Conic Sections in Standard Form STA: CA.CACS.MTH.97.AII.11.7
LOC: MTH.C.10.09.04.004 TOP: 12-6 Identifying Conic Sections
MSC: DOK 1
82. **ANS:**

The standard form of an ellipse, where \( a > b \), can be written as

\[
\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1, \text{ where the major axis is horizontal, or}
\]

\[
\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1, \text{ where the major axis is vertical.}
\]

\[
\frac{(x+9)^2}{5^2} + \frac{(y-6)^2}{2^2} = 1 \text{ is an ellipse with a horizontal axis.}
\]

**PTS:** 1  **DIF:** Advanced  **REF:** 175b8dda-4683-11df-9c7d-001185f0d2ea

**OBJ:** 12-6.1 Identifying Conic Sections in Standard Form  **STA:** CA.CACS.MTH.97.AII.17.0

**LOC:** MTH.C.10.09.04.004  **TOP:** 12-6 Identifying Conic Sections

**MSC:** DOK 1

83. **ANS:**

The standard form of an ellipse, where \( a > b \), can be written as

\[
\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1, \text{ where the major axis is horizontal, or}
\]

\[
\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1, \text{ where the major axis is vertical.}
\]

\[
\frac{(x+6)^2}{8^2} + \frac{(y+5)^2}{5^2} = 1 \text{ is an ellipse with a horizontal axis.}
\]

**PTS:** 1  **DIF:** Advanced  **REF:** 175b8dda-4683-11df-9c7d-001185f0d2ea

**OBJ:** 12-6.1 Identifying Conic Sections in Standard Form  **STA:** CA.CACS.MTH.97.AII.17.0

**LOC:** MTH.C.10.09.04.004  **TOP:** 12-6 Identifying Conic Sections

**MSC:** DOK 1
The general form for a conic section is \( Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \).

\[ A = 7, \ B = -8, \ C = -4 \]

Identify the values for \( A, B, \) and \( C \).

\[ B^2 - 4AC = (-8)^2 - 4(7)(-4) \]

Substitute into \( B^2 - 4AC \).

\[ B^2 - 4AC = 176 \]

Simplify.

Because \( B^2 - 4AC > 0 \), the equation represents a hyperbola.

The graph of the first equation is a circle, and the graph of the second equation is a parabola. There may be as many as four points of intersection.

**Step 1** It is simplest to solve for \( x^2 \) because both equations have \( x^2 \)-terms.

\( x^2 = 2y + 10 \)

Solve for \( x^2 \) in the second equation.

**Step 2** Use substitution.

\( (2y + 10) + y^2 = 25 \)

Substitute this value into the first equation.

\( y^2 + 2y - 15 = 0 \)

Simplify, and set equal to 0.

\( (y - 3)(y + 5) = 0 \)

Factor.

\( y = 3 \) or \( y = -5 \)

**Step 3** Substitute 3 and \(-5\) into \( x^2 = 2y + 10 \) to find values for \( x \).

\( x^2 = 2(3) + 10 = 16 \)

\( x = \pm 4 \)

\( (4,3) \) and \((-4,3)\) are solutions.

\( x^2 = 2(-5) + 10 = 0 \)

\( (0,-5) \) is a solution.

The solution set of the system is \{\( (4,3), (-4,3), (0,-5) \}\).
86. **ANS:**
The system has no solution.
The graph of the first equation is an ellipse, and the graph of the second equation is a hyperbola. There may be as many as four points of intersection.

**Step 1** Eliminate $x$.

\[
\begin{align*}
\quad & 4x^2 + 9y^2 = 36 \\
- & (4x^2 - 25y^2 = 100) \\
\hline
\quad & 34y^2 = -64 \\
\quad & y^2 = -\frac{32}{17}
\end{align*}
\]

Since the square of a real number cannot be negative, the system has no solution.

**Check** The graph supports that there are zero points of intersection.
87. \( \text{ANS:} \)
\[
\left\{ -1, \frac{\sqrt{6}}{2} \right\}, \left\{ -1, -\frac{\sqrt{6}}{2} \right\}
\]
A sketch of the graphs show that there will be two points of intersection.

\[
\begin{align*}
solve \quad & \begin{cases}
  x^2 + 4y^2 = 7 \\
  2y^2 - x = 4
\end{cases}
\end{align*}
\]

**Step 1** Solve for \( x \).

\[
\begin{align*}
2x - 4y^2 &= -8 \\
\hline
x^2 + 4y^2 &= 7 \\
+2x - 4y^2 &= -8 \\
x^2 + 2x &= -1 \\
x^2 + 2x + 1 &= 0 \\
(x + 1)^2 &= 0 \\
x &= -1
\end{align*}
\]

Solve for \( x \).

**Step 2** Solve for \( y \).

\[
\begin{align*}
2y^2 - (-1) &= 4 \\
2y^2 &= 3 \\
y &= \pm \sqrt{\frac{3}{2}} = \pm \frac{\sqrt{6}}{2}
\end{align*}
\]

Solve for \( y \).
88. **ANS:**
   Population: All mule deer
   Sample: The deer in the herd being sampled

   PTS: 1  DIF: Basic  REF: 9074c1b9-6ab2-11e0-9c90-001185f0d2ea
   NAT: NT.CCSS.MTH.10.9-12.S.IC.1  TOP: 8-2 Data Gathering
   MSC: DOK 2

89. **ANS:**
   self-selected

   PTS: 1  DIF: Basic  REF: 9098ac1e-6ab2-11e0-9c90-001185f0d2ea
   OBJ: 8-5.1 Classifying a Sample  TOP: 8-5 Sampling Distributions
   KEY: sampling methods  MSC: DOK 2

90. **ANS:**
   36 days

   PTS: 1  DIF: Average  REF: 9080d490-6ab2-11e0-9c90-001185f0d2ea
   OBJ: 8-2.3 Using Data to Make Predictions  NAT: NT.CCSS.MTH.10.9-12.S.IC.1
   TOP: 8-2 Data Gathering  KEY: sample | prediction from a sample
   MSC: DOK 3