Name: ________________________________ Date: ___________ Period:______

CHAPTER 5: Probability: What Are The Chances?

CHAPTER INTRODUCTION:

Now that we have learned how to collect data and how to analyze it graphically and numerically, we turn our study to ________________________, the mathematics of chance. Probability is the basis for the fourth and final theme next semester, _________________. The next three chapters will provide you with the background in probability necessary to perform and understand the inferential methods we'll see later in the course.

In this chapter, you will learn the definition of probability as a long-term __________________ frequency. You will study how to use ________________________ to answer probability questions as well as some basic rules to calculate probabilities of events. You will also learn two concepts that will reappear later in our studies: _________________________ probability and ______________________. Many of the ideas and methods you will learn in this chapter may be familiar to you. When it comes to statistics, your goal with probability is to be able to answer the question, "What would happen if we did this many times?" so you can make an informed statistical ______________________.

5.1: *Randomness, Probability, and Simulation*

SECTION INTRODUCTION:

This section introduces the basic definition of ______________________ as a long-term relative frequency. There are a lot of common misconceptions about probability. Several are discussed in this section. Be sure to avoid falling into these common myths! The last topic in this section addresses the use of __________________________ to estimate probabilities. Simulation is a powerful tool for modeling chance behavior that can be used to illustrate many of the inference ideas you'll study later in the course.

5.1-KEY VOCABULARY AND CONCEPTS: law of large numbers, probability, simulation

5.1- CONCEPT 1: The Idea of Probability (Page 282-288)

When we observe _______________ behavior over a long series of repetitions, a useful fact emerges. While chance behavior is unpredictable in the short term, a regular and predictable pattern becomes evident in the ________ ______. The law of _____________ ______________ tells us that as we observe more and more repetitions of chance behavior, the ______________ of times a specific outcome occurs will "settle down" around a single value. This long-term proportion is the _____________________ of the outcome occurring. The probability of an event is always described as a value between _____ and _____, inclusive. With 0 representing it is _____________________ for the event to occur and 1 representing it is _________________ the event will occur.

5.1 EXAMPLE 1: The probability of drawing a jack, queen, or king from a standard deck of playing cards is approximately 0.23 (12/52). Does this mean if we repeatedly draw a card, replace it, shuffle, and draw again 100 times that we will draw a jack, queen, or king 23 times? Why or why not?

5.1- CONCEPT 2: Simulation (Page 289-292)

In this section, we learn how we can use _________________________ to estimate the probability of an event occurring. The four-step process can be used to perform a simulation by identifying the ______________ of interest about the chance process, _________________________ how to use a chance device to imitate a repetition of the chance behavior, performing many ______________________ of the simulation, and using the results of the simulation to ______________ the original question. While simulations don't provide exact theoretical probabilities, the use of random numbers and other chance devices to imitate chance behavior can be a useful tool for ________________________ the likelihood of events.

Tools used in simulations: ________, ________, __________, _____________, _____________ __________ ________, or _________________________ just to name a few.

5.1 EXAMPLE 2: Suppose we are interested in estimating the likelihood of a couple's having a girl among their first four children. Assume that girls and boys are equally likely to occur on any birth and that your coin is fair. Describe a simulation to estimate this probability.

5.1 EXAMPLE 3: A popular airline knows that, in general, 95% of individuals who purchase a ticket for a 10-seat commuter flight actually show up for the flight. In an effort to ensure a full flight, the airline sells 12 tickets for each flight. Design and carry out a simulation to estimate the probability that the flight will be overbooked, that is, more passengers show up than there are seats on the flight.

5.2: ***Probability Rules***

SECTION INTRODUCTION:

Now that you have the basic idea of probability down, you will learn how to describe probability _____________ and use probability rules to ____________________ the likelihood of events. You will learn how to organize information in _______-______ ___________ and Venn diagrams to help in determining probabilities. Understanding probability is important for understanding _________________. Make sure you are comfortable with the definitions and rules in this section as it will make your study of probability much easier!

5.2-KEY VOCABULARY AND CONCEPTS: sample space (*S*), probability model, event, complement, mutually exclusive (disjoint), general addition rule, intersection, union

5.2- CONCEPT 1: Probability Models and the Basic Rules of Probability (Page 299-302)

Chance behavior can be described using a probability _____________. The model provides two pieces of information: a list of possible outcomes (______________ ____________) and the likelihood of each outcome. By describing chance behavior with a probability model, we can find the probability of an ____________--a particular outcome or collection of outcomes. Probability models must obey some basic rules of probability:

- For any event *A*, _______________________
- If *S* is the sample space in a probability model, ____________
- In the case of equally likely outcomes,

  ____
- For any even *A* there exists a complement, _________________________
- If *A* and *B* are mutually exclusive, _________________________________

After this section, you should be able to describe a probability model for chance behavior and apply the basic probability rules to answer questions about events.

5.2 EXAMPLE 1: Consider drawing a card from a shuffled fair deck of 52 playing cards.

(a) How many possible outcomes are there in the sample space for the chance process? What's the probability of each outcome?

Define the following events: *A*=the card drawn is an Ace, *B*= the card drawn is a heart

(b) Find *P*(*A*) and *P*(*B*).

(c) What is $P(A^c)$?

(d) Are the events *A* and *B* mutually exclusive? Why or why not?

5.2- CONCEPT 2: Two-Way Tables and Venn Diagrams (Page 303-308)

Often we will need to find probabilities involving ______ events. In these cases, it may be helpful to organize and display the sample space using a two-way table or Venn diagram. This can be especially helpful when two events are ______ mutually exclusive. When dealing with two events *A* and *B*, it is important to be able to describe the ____________ (or collection of all outcomes in *A*, *B*, or both) and the _______________________ (the collection of outcomes in both *A* and *B*). The general addition rule expands upon the basic rules presented in this section to help us find the probability of two events that are NOT mutually exclusive.

This leads us to the general addition rule: ____

If *A* and *B* are mutually exclusive, then we get: ____

5.2 EXAMPLE 2: consider drawing a card from a shuffled fair deck of 52 playing cards. Define the following events: *A*=the card drawn is an Ace, *B*=the card drawn is a heart.

(a) Use a two way table to display the sample space.

(b) Use a Venn diagram to display the sample space.

(c) Find $P(A \cup B)$. Show your work.

5.3: ***Conditional Probability and Independence***

SECTION INTRODUCTION:

Two important concepts are introduced in this section: ___________________ _____________ and _________________________. These concepts will reappear throughout the remainder of you studies in statistics, so it is very important that you understand what they mean. This section will also introduce you to several rules for calculating probabilities: the general ___________________ rule, the multiplication rule for ______________________ events, and the ____________________ probability formula. Not only do you want to know how to use these rules, but also when. As you perform probability calculations, make sure you can _____________ why you are using a particular rule.

5.3-KEY VOCABULARY AND CONCEPTS: conditional probability, independent, tree diagram, general multiplication rule, multiplication rule for independent events, conditional probability formula

5.3- CONCEPT 1: Conditional Probability and Independence (Page 312-316)

A ______________________ probability describes the chance that an even will occur given that another even is already known to have happened. To note that we are dealing with a conditional probability, we use the symbol of a vertical line to mean "given that".

For example, suppose we draw a card from a shuffled deck of 52 playing cards. We could write "the probability that the card is an ace given that it is a red cars" as ______________________.

Building on the concept of conditional probabilities, we can say that when knowing that one even has occurred has ____ ___________ on the probability of another event occurring, the events are said to be ______________________. That is, events *A* and *B* are independent if __________________ and _________________.

<u>5.3 EXAMPLE 1:</u> Is there a relationship between gender and candy preference? Suppose 200 high school students were asked to complete a survey about their favorite candies. The table below shows the gender of each student and their favorite candy.

	Male	Female	Total
Skittles	80	60	140
M & M's	40	20	60
Total	120	80	200

Define *A* to be the event that a randomly selected student is *male* and *B* to be the event that a randomly selected student likes *Skittles*. Are the events *A* and *B* independent? Justify your answer.

<u>5.3 EXAMPLE 2:</u> Students at the University of New Harmony received 10,000 course grades last semester. The two-way table below breaks down these grades by which school university taught the course. The schools are Liberal Arts, Engineering and Physical Sciences, and Health and Human Services.

		Grade Level	
School	**A**	**B**	**Below B**
Liberal Arts	2,142	1,890	2,268
Engineering and Physical Sciences	368	432	800
Health and Human Services	882	630	588

The table is based closely on grade distributions at an actual university, simplified a bit for clarity. College grades tend to be lower in engineering and the physical sciences (EPS) than in liberal arts and social sciences (which includes Health and Human Services). Consider the two events *E*= the grade outcomes from an EPS course, and *L*=the grade is lower than a B.

(a) Find $P(L)$. Interpret this probability in context.

(b) Find $P(E|L)$ and $P(L|E)$.

(c) Are the events *E* and *L* independent? Explain.

5.3- CONCEPT 2: Tree Diagrams and the Multiplication Rule (Page 317-323)

When chance behavior involves a ______________________ of events, we can model it using a tree diagram. A tree diagram provides a branch for each _____________________ of an event along with the associated _________________________ of those outcomes. Successive branches represent particular sequences of outcomes. To find the probability of an event, we ___________________ the probabilities on the branches that make up the event.

This leads us to the general multiplication rule: __________________________________
If *A* and *B* are independent, then we get: _______________________________________

5.3 EXAMPLE 3: A study of high school seniors in three school districts (Lakeville, Sheboygan, and Omaha) was conducted to determine the trends in AP mathematics courses: Calculus or Statistics. 42% of students in the study came from Lakeville, 37% came from Sheboygan, and the rest came from Omaha. In Lakeville, 64% of seniors took Statistics and the rest took Calculus. 58% of seniors from Sheboygan and 49% of seniors in Omaha took Statistics while the rest took Calculus in each district. No seniors took both Statistics and Calculus. Describe this situation using a tree diagram and find the probability that a randomly selected student took statistics.

5.3- CONCEPT 3: Calculating Conditional Probabilities (Page 324-328)

By rearranging the terms in the general multiplication rule, we can determine a rule for conditional probabilities:

____

Most conditional probabilities can be determined using a two-way table, Venn diagram, or tree diagram. However, the formula can also be useful if you know the appropriate probabilities for the situation.

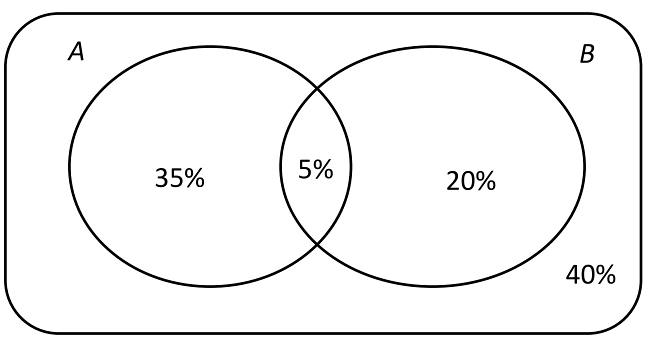
5.3 EXAMPLE 4: Consider the situation from Example-3.
Find *P*(student is from Lakeville | they took Statistics).

<u>5.3 EXAMPLE 5:</u> Below is a table classifying U.S. households according to the types of phones they used. Each entry gives you the percentage of U.S. households included in the survey fulfilling the category.

	Cell Phone	**No Cell Phone**	**Total**
Landline	0.60	0.18	0.78
No Landline	0.20	0.02	0.22
Total	0.80	0.20	1.00

What is the probability that a randomly selected household with a landline also has a cell phone?

<u>5.3 EXAMPLE 6:</u> Suppose residents of a large apartment complex were classified based on two events: *A*=reads *USA Today*, and *B*=reads the *New York Times*. Use the provided Venn diagram below to answer the question.



What is the probability that a randomly selected resident who read *USA Today* also reads the *New York Times*?

CHAPTER SUMMARY:

Probability describes the long-term behavior of chance processes. Since chance occurrences display patterns of regularity after ____________ repetitions, we can use the rules of probability to determine the likelihood of observing particular results. At this point, you should be comfortable with the basic definition and rules of probability. In the next two chapters, you will study some ________________ concepts in probability so we can build the foundation necessary for statistical ________________.

Note that the AP exam may contain several questions about the probability of particular events. Make sure you understand ______ and ___________ to apply each formula. More importantly, make sure you show your work and use good notation when calculating probabilities so anyone reading your response understands exactly how you arrived at your answer.