Lesson Preview

What You’ll Learn

To find missing measures of similar figures

To use similar figures when measuring indirectly

. . . And Why

To apply proportions when finding distances represented on maps, as in Example 3

Check Skills You’ll Need (For help, go to the Skills Handbook and Lesson 4-1.)

Simplify each ratio.

1. \( \frac{16}{16} \)
2. \( \frac{21}{21} \)
3. \( \frac{54}{54} \)

Solve each proportion.

4. \( \frac{17}{17} = \frac{25}{25} \)
5. \( \frac{27}{27} = \frac{25}{25} \)
6. \( \frac{12}{12} = \frac{12}{12} \)

7. \( \frac{20}{20} = \frac{10}{10} \)
8. \( \frac{15}{15} = \frac{15}{15} \)
9. \( \frac{24}{24} = \frac{12}{12} \)

New Vocabulary

• similar figures
• scale drawing
• scale

Investigation: Proportions in Triangles

The figure below shows \( \triangle ABC \) and \( \triangle DCE \).

\[
\begin{align*}
AB &= 8.5 \text{ cm},
CA &= 7.6 \text{ cm},
CB &= 5.6 \text{ cm},
CD &= 3.5 \text{ cm},
CE &= 2.6 \text{ cm},
DE &= 3.9 \text{ cm}
\end{align*}
\]

1. Measure \( AB \), \( CA \), \( CB \), \( CD \), \( CE \), and \( DE \) using a metric ruler. See above.

2. Find each ratio.
   a. \( \frac{AB}{CB} = \frac{2.5}{3.5} = 0.71 \)
   b. \( \frac{CE}{CB} = \frac{2.6}{3.5} = 0.74 \)
   c. \( \frac{CD}{CA} = \frac{3.5}{7.6} = 0.46 \)

3. Tell whether each statement is true.
   a. \( \frac{AB}{CB} = \frac{CE}{CB} \) true
   b. \( \frac{CD}{CA} = \frac{CE}{CB} \) true
   c. \( \frac{AB}{DE} = \frac{CA}{CD} \) true

4. Using the lengths you have measured, write two ratios that equal \( \frac{CE}{CB} \).
   \( \frac{CA}{CD} \), \( \frac{AB}{DE} \)

Lesson 4-2 Proportions and Similar Figures 189
In the diagram below, \( \triangle ABC \) and \( \triangle FGH \) are similar. **Similar figures** have the same shape but not necessarily the same size. The symbol \( \sim \) means “is similar to.”

In similar triangles, corresponding angles are congruent and corresponding sides are in proportion. The order of the letters indicates the corresponding angles. If \( \triangle ABC \sim \triangle FGH \), then the following is true.

**Finding the Length of a Side**

Geometry In the figure below, \( \triangle ABC \sim \triangle DFE \). Find \( DE \).

**EXAMPLE**

Sketch

- \( A \): 21 cm
- \( B \): 15 cm
- \( C \): 18 cm
- \( D \): 10 cm
- \( E \): \( x \) cm
- \( F \): 12 cm

**Relate** \( \frac{AB}{DF} = \frac{AC}{DE} \)

**Define** Let \( x = DE \).

**Write** \( \frac{15}{10} = \frac{21}{x} \)

\[ 15x = 210 \]

\[ x = 14 \]

\( DE \) is 14 cm.

**Check Understanding**

In the figure below, \( \triangle FGH \sim \triangle KLM \). Find \( LM \).

**EXAMPLE**

Sketch

- \( F \): 32 cm
- \( G \): 20 cm
- \( H \): 24 cm
- \( K \): \( x \) cm
- \( L \): \( y \) cm
- \( M \): \( z \) cm

**Relate** \( \frac{FG}{KL} = \frac{GH}{LM} \)

**Write** \( \frac{32}{20} = \frac{24}{LM} \)

**Define** Let \( y = LM \).

**Write** \( \frac{32}{20} = \frac{24}{y} \)

\[ 32y = 480 \]

\[ y = 15 \]

\( LM \) is 15 cm.

**Need Help?**

Congruent angles have equal measures. The symbol \( \cong \) means “is congruent to.”

**Additional Examples**

In the figure below, \( \triangle ABC \sim \triangle DEF \). Find \( AB \).

**EXAMPLE**

Sketch

- \( A \): 10 mm
- \( B \): 12 mm
- \( C \): 9 mm
- \( D \): 8 mm
- \( E \): \( x \) mm
- \( F \): 12 mm

**Relate** \( \frac{AB}{DF} = \frac{AC}{DE} \)

**Define** Let \( x = DE \).

**Write** \( \frac{10}{12} = \frac{9}{x} \)

\[ 10x = 108 \]

\[ x = 10.8 \]

\( DE \) is 10.8 mm.

**Check Understanding**

In the figure below, \( \triangle FGH \sim \triangle KLM \). Find \( LM \).

**Advanced Learners** Challenge students to write as many proportions as possible that can be used to solve the problem in Example 1.

**English Learners** See note on page 190.

**Inclusion** See note on page 191.

**Reaching All Students**

**Below Level** Suggest that before students write a proportion involving similar figures, they point to and say to themselves the names of the corresponding sides.
You can use proportions to find the dimensions of objects that are difficult to measure directly.

**Applying Similarity**

**Indirect Measurement** A tree casts a shadow 7.5 ft long. A woman 5 ft tall casts a shadow 3 ft long. The triangle shown for the tree and its shadow is similar to the triangle shown for the woman and her shadow. How tall is the tree?

\[
\frac{3}{7.5} = \frac{5}{x}
\]

Corresponding sides of similar figures are in proportion.

\[
3x = 7.5 \cdot 5
\]

Write cross products.

\[
3x = 37.5
\]

Simplify.

\[
x = 12.5
\]

Divide each side by 3.

- The tree is 12.5 ft tall.

**Check Understanding**

a. A tree casts a 26-ft shadow. A boy standing nearby casts a 12-ft shadow. His height is 4.5 ft. How tall is the tree? 9.75 ft

b. A house casts a 56-ft shadow. A girl standing nearby casts a 7.2-ft shadow. Her height is 5.4 ft. What is the height of the house? 42 ft

A **scale drawing** is an enlarged or reduced drawing that is similar to an actual object or place. Floor plans, blueprints, and maps are all examples of scale drawings. The ratio of a distance in the drawing to the corresponding actual distance is the **scale** of the drawing.

**Finding Distances on Maps**

The scale of the map at the left is 1 inch : 10 miles. Approximately how far is it from Valkaria to Wabasso?

\[
\frac{\text{map distance}}{\text{actual distance}} = \frac{1}{10}
\]

- Measure the map distance.
- Write a proportion.
- Write cross products.
- Simplify.

- Wabasso is about 17.5 mi from Valkaria.

**Check Understanding**

a. On the map above, measure the map distance from Grant to Gifford. Find the actual distance. about 21 mi

b. **Critical Thinking** If another map showed the distance from Valkaria to Wabasso but had a scale of 1 inch : 5 miles, what would the map distance be between the two locations? 3.5 in.
The figures in each pair are similar. Identify the corresponding sides and angles.

1. \( \triangle ABC \sim \triangle PQR \)
   - \( AB \sim PQ \), \( BC \sim QR \)
   - \( \angle A \sim \angle P \), \( \angle B \sim \angle Q \), \( \angle C \sim \angle R \)

2. \( \triangle DEF \sim \triangle HJK \)
   - \( DE \sim HJ \), \( EF \sim HK \)
   - \( \angle D \sim \angle H \), \( \angle E \sim \angle J \), \( \angle F \sim \angle K \)

The child in the figure is 3 ft tall.

9. How tall is the tree? 4.8 ft

10. The cat casts an 18-in. shadow. How tall is the cat? 12 in.

Example 3 (page 191)

The scale of a map is 1 in. : 17.5 mi. Find the actual distance corresponding to each map distance.

11. 5 in. 87.5 mi
12. 8.3 in. 145.25 mi
13. 18.6 in. 325.5 mi
14. 20 in. 350 mi

For more practice, see Extra Practice.
15. a. Use a ruler and the map at the left. Find the distance from each town to the others. See margin.
   b. A student lives halfway between Lincoln and San Paulo and takes the shortest route to school in Duncanville. How far does the student travel each day to school? 26 mi
16. The actual distance between two towns is 28 km. Suppose you measure the distance on your map and find that it is 3.5 cm. What is the scale of your map? 1 cm : 8 km
17. 1 in. : 2 ft 18. 1 in. : 3 ft 19. 1 in. : 4 ft 20. 1 in. : 5 ft 4 in. by 6 in. 21. Two rectangles are similar. The first is 4 in. wide and 15 in. long. The second is 2 m by 3 in. What are the actual dimensions of the room? 8 ft by 12 ft room. 22. Architecture A blueprint scale is 1 in. : 12 ft. On the plan, the room measures 2.5 in. by 3 in. What are the actual dimensions of the room? 22.5 ft by 27 ft
23. Error Analysis The two figures are similar. Robert uses the proportion $\frac{GH}{PQ} = \frac{GR}{RQ}$ to find $RQ$.
   a. What is Robert’s error?
   b. What proportion should he have used? $\frac{GH}{RL} = \frac{GR}{RQ}$
24. What is the scale of the drawing? 1 in. : 12 ft
25. What are the actual dimensions of the kitchen? 9 ft by 12 ft
26. Find the actual width of the doorways that lead into the kitchen and the dining room. 3 ft
27. Find the actual area of the dining room. 216 ft²
28. Can a table 7 ft long and 4 ft wide fit into the narrower section of the dining room? Explain your answer. Yes; because it is 6 ft wide and 9 ft long
29. Two rectangles are similar. One is 5 cm by 12 cm. The longer side of the second rectangle is 8 cm greater than twice its shorter side. Find its length and width. 48 cm long by 20 cm wide
30. Geometry Rectangle $ABCD$ is similar to rectangle $KLMN$.
   a. What is the width $w$ of rectangle $KLMN$? 6 m
   b. What is the perimeter of each rectangle? 6 m, 18 m
   c. Is the ratio of the perimeters of the rectangles (small : large) equal to the ratio of corresponding sides? Explain.
   d. What is the area of each rectangle?
   e. Critical Thinking Find the ratio of the areas (small : large). Explain how the ratio of the areas is related to the ratio of the corresponding sides.
   Answers may vary. Sample: The area ratio is the square of the side ratio.
31. Open-Ended Give some examples of similar figures found in everyday life. See margin.
32. **Writing** Are the two cubes similar? Explain your answer.

b. Explain how the ratio of volumes (small : large) is related to the ratio of their sides (small : large).

c. If the ratio of the sides of the two cubes is 3 : 1, what is the ratio of their volumes?  
   a-c See margin.

33. **Geometry** The perimeter of a triangle with sides \( a \), \( b \), and \( c \) is 24 cm. Side \( a \) is 2 cm longer than side \( b \). The ratio of the lengths of sides \( b \) and \( c \) is 3 : 5. What are the lengths of the three sides of the triangle? \( a \neq 8 \), \( b \neq 6 \), \( c \neq 10 \)

34. The state of Alabama is about 335 mi long and 210 mi wide. What scale would you use to draw a map of Alabama on an in.-by-11 in. paper to make the map as large as possible?  
   about 1 in. : 24.7 mi

35. **Astronomy** You can block out the moon by holding a coin up at a distance from your eye that is 110 times the diameter of the coin. Using similar figures, what is how far away is it?  
   \( 400,400 \) km

36. **Geometry** In the figure at the right, \( \triangle ABC \sim \triangle ADE \).  
   a. Substitute values from the diagram into the following proportion \( \frac{AD}{AP} = \frac{DE}{DP} \).  
      \( \frac{8}{x + 2} = \frac{5}{3} \)  
      \( \text{Hint: } AB = AD + DB \).
   b. Solve the proportion for \( x \). 3.2
   c. Find the length of \( AB \). 11.2 in.
   d. What is the area of \( \triangle ABC \)? 39.2 in.²

37. In the figure at the right, \( \triangle ABC \sim \triangle XYZ \). Which proportion is incorrect? B

a. \( \frac{AB}{AC} = \frac{XY}{XZ} \)

b. \( \frac{AB}{BC} = \frac{XY}{XZ} \)

c. \( \frac{BC}{AC} = \frac{YZ}{XZ} \)

d. \( \frac{AC}{XZ} = \frac{YZ}{XZ} \)
Lesson 4-2
Proportions and Similar Figures

38. A map of Kentucky is drawn with a scale of 1 cm : 11 km. The map distance between Louisville and Bowling Green is 14.5 cm. Which is the best estimate of the actual distance? 

I. 1.3 km
G. 14 km
H. 100 km
I. 160 km

Short Response
39. You can paint a 6 ft-by-5 ft rectangular wall using 0.5 gallon of paint. How many gallons of paint will you need to cover a 10 ft-by-12 ft wall? Show your work. See margin.

Extended Response
40. Leonardo da Vinci’s famous painting the Mona Lisa measures 77.5 cm by 55 cm. See margin.

a. Explain how you know that a 16 cm-by-12 cm reproduction postcard is NOT similar to the original painting.

b. What dimensions would make a postcard similar to the original painting? Approximate to the nearest tenth. Show your work or explain how you found your answer.

Mixed Review

Lesson 4-1
Solve each proportion.
41. \( \frac{x}{4} = \frac{4}{5} \)
42. \( \frac{p}{7} = \frac{3}{10} \)
43. \( \frac{y}{8} = \frac{7}{20} \)
44. \( \frac{12}{30} = \frac{16}{v} \)

Lesson 3-3
Solve.
45. \( 5b < -20 \)
46. \( \frac{1}{3}x = 4 \) OR \( 7 \)
47. \( -3m > 12 \)
48. \( \frac{7}{3}h < 1 \)

40a.

b. Explain how you know that a 16 cm-by-12 cm reproduction postcard is NOT similar to the original painting.

b. What dimensions would make a postcard similar to the original painting? Approximate to the nearest tenth. Show your work or explain how you found your answer.

Checkpoint Quiz 1
Lessons 4-1 through 4-2

1. Complete the statement 2 days = ___ minutes. 2880
2. Write $48 for 8 hours as a unit rate. $6.00/h

Solve each proportion.
3. \( \frac{x}{6} = \frac{7}{4} \)
4. \( \frac{y}{k} = -\frac{13}{20} \)
5. \( \frac{3}{5} = \frac{y + 1}{9} \)

6. You are riding your bicycle. It takes you 12 min to go 2.5 mi. If you continue traveling at the same rate, how long will it take you to go 7 mi? 33.6 min

The figures in each pair are similar. Find the missing length.
7. \( \begin{array}{c}
5 \text{ cm} \\
8 \text{ cm} \\
3.125 \text{ cm}
\end{array} \)
8. \( \begin{array}{c}
4.5 \text{ ft} \\
6 \text{ ft} \\
3 \text{ ft} \\
y
\end{array} \)

9. A 3.5-ft child casts a 60-in. shadow. She is standing next to a telephone pole that casts a 50-ft shadow. How tall is the telephone pole? 55 ft

10. The scale of a map is 3 in. : 20 mi. Find the actual distance if the map distance between two towns is 5.5 in. \( 36\frac{2}{3} \) mi

Lesson 4-2
Proportions and Similar Figures

39. [2] smaller area: \( 6 \cdot 5 = 30; 30 \text{ ft}^2 \); larger area; \( 10 \cdot 12 = 120; 120 \text{ ft}^2 \);

Two gallons of paint should cover a 10 ft \( \times \) 20 ft wall.

[1] incorrect calculation for one area and proportion solved correctly OR correct area calculations but proportion set up incorrectly

[2] incorrect proportions solved correctly

[1] correct answer with no work shown